A HYBRID COMPUTER TECHNIQUE FOR MEASURING HUMAN DESCRIBING FUNCTIONS AND REMNANT IN CLOSED-LOOP TRACKING TASKS

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THESIS

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FOR
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REMNANT IN CLOSED-LOOP TRACKING TASKS

by

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ABSTRACT

The measurement of the human describing function and remnant in a compensatory tracking task is undertaken. These measurements are obtained through the application of the fast Fourier transform technique on a hybrid (analog-digital) computer. This method processes the data in real time with minimal core storage and the results are available immediately upon completion of the tracking run.

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TABLE OF SYMBOLS

A_{f_k}	REAL PART OF FOURIER TRANSFORM OF f(t) AT FREQUENCY ω_k
B_{f_k}	IMAGINARY PART OF FOURIER TRANSFORM OF f(t) AT FREQUENCY, ω_k
c(t)	OUTPUT SIGNAL
e(t)	ERROR SIGNAL
f(t)	ARBITRARY SIGNAL
$F(j\omega)$	FOURIER TRANSFORM OF f(t)
$\bar{F}(j\omega)$	COMPLEX CONJUGATE OF $F(j\omega)$
F(n)	FOURIER COEFFICIENT FOR A PERIODIC SIGNAL f(t)
$\bar{F}(n)$	COMPLEX CONJUGATE OF F(n)
H(s)	SYSTEM TRANSFER FUNCTION
i(t)	INPUT SIGNAL
$i_T(t)$	INPUT SIGNAL OF FINITE DURATION
n(t)	REMNANT SIGNAL
p(t)	TOTAL OPERATOR RESPONSE SIGNAL, p'(t) + n(t)
p'(t)	LINEAR OPERATOR RESPONSE SIGNAL
T	PERIOD OF TOTAL RUN LENGTH
T_k	PERIOD FOR FREQUENCY, ω_k
$Y_c(j\omega)$	CONTROLLED ELEMENT TRANSFER FUNCTION
$Y_p(j\omega)$	HUMAN DESCRIBING FUNCTION
$\phi_{ff}(\tau)$	AUTOCORRELATION FUNCTION OF f(t)
$\phi_{f_1 f_2}(\tau)$	CROSSCORRELATION FUNCTION OF $f_1(t)$ and $f_2(t)$
$\Phi_{ff}(n)$	POWER SPECTRAL DENSITY OF PERIODIC SIGNAL, f(t)

$\Phi_{f_1 f_2}(n)$ CROSS-POWER SPECTRAL DENSITY OF PERIODIC SIGNALS $f_1(t)$ and $f_2(t)$

$\Phi_{ff}(\omega)$ POWER (ENERGY) SPECTRAL DENSITY OF $f(t)$

$\Phi_{f_1 f_2}(\omega)$ CROSS-POWER (ENERGY) SPECTRAL DENSITY OF $f_1(t)$ and $f_2(t)$

ω_h FREQUENCIES OTHER THAN THOSE IN THE INPUT SINUSOIDS

ω_k FREQUENCIES OF INPUT SINUSOIDS

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I. INTRODUCTION

A. BACKGROUND

Many of the important tasks performed by pilots are akin to those performed by linear servomechanisms. In situations such as this, the pilot can be modeled by a set of constant-coefficient linear differential equations. In the frequency domain, such a model is often referred to as a "human pilot describing function." The term "describing function" is preferred to "transfer function" to emphasize the fact that this pilot model is approximating a nonlinear element and is valid only for the particular inputs, system dynamics and task at hand.

The pilot-describing function is useful in studying two classes of problems. First, the describing functions measured in the piloted simulation of a given aircraft and task can be utilized in the subsequent stability and control analysis of this aircraft. Once the pilot's describing function for a particular task has been measured, he can be analytically replaced by his describing function in the analyses normally associated with the study of linear feedback systems. Second, actual measurement of pilot describing functions in ground simulation or in flight tests can be used to determine how a particular aircraft or flight task affects pilot behavior. Knowledge of pilot describing functions consequently provides valuable information for

the aircraft designer.

It is the problem of human describing function measurement which forms the basis of this research. The hybrid computer program which has resulted can be utilized in virtually any study involving human behavior in compensatory tracking tasks.

B. COMPENSATORY TASKS AND QUASI-LINEARIZATION

The compensatory tracking task, shown in Figure 1, assumes that the error signal is the only information that the operator is receiving. In this study the operator attempts to minimize a visual error signal by using a hand-operated controller. Tracking situations such as this are often encountered in aircraft flight control; e.g., a pilot attempting to maintain some desired pitch attitude in the presence of atmospheric turbulence. It has been shown that in tasks such as this, the operator is nonlinear and time variant in behavior. He may, however, be successfully modeled in a quasi-linear fashion [Ref. 1]. This quasi-linearization implies that his response to visual stimulation is largely linear and time invariant; i.e., his dynamics are largely those of constant-coefficient linear differential equations. To account for nonlinear and/or time-varying behavior, the model includes a remnant signal as shown in Figure 2. The remnant is that portion of the operator's output which is not linearly correlated with the input. The human operator model thus consists of a linear describing function, $Y_p(j\omega)$, determined from the quasi-linear

analysis, and a remnant, n(t).

It should be noted that this quasi-linear model is of little use if the remnant is relatively large, since then the operator's behavior is predominantly nonlinear and/or time varying.

To determine the human operator model it is necessary to calculate the linear describing function $Y_p(j\omega)$, and the remnant power spectral density, $\Phi_{nn}(\omega)$, from physical measurements of signals of finite duration in a laboratory experiment. The input signal must appear to the operator to be random, although it need not be truly random, and the operator must be well trained; i.e., not undergoing adaptation or learning [Ref. 2].

In order to measure the human describing function and remnant, one of three techniques can be employed. These are the direct Fourier analysis of the system signals, the use of crosscorrelation methods, and a model optimization technique. These methods are discussed in Ref. 3.

If, as done here, direct Fourier analysis or cross-correlation methods are employed in measuring the human describing function and remnant, then the concept of spectral analysis must be introduced. The next section is devoted to this topic. A more thorough treatment can be found in Ref. 4, Appendix D.

II. SPECTRAL ANALYSIS

A. PERIODIC SIGNALS

A periodic signal, $f(t)$, with a fundamental frequency ω_1 and period T, satisfying the Dirichlet conditions [Ref. 4, p. 579], may be represented by a Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} F(n) e^{jn\omega_1 t} ,$$

where $$F(n) = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn\omega_1 t} dt .$$

The autocorrelation function for the above periodic signal is defined as

$$\phi_{ff}(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} f(t) f(t+\tau) dt .$$

This can be written

$$\phi_{ff}(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} f(t) \sum_{n=-\infty}^{\infty} F(n) e^{jn\omega_1 (t+\tau)} dt$$

and is equivalent to

$$\phi_{ff}(\tau) = \left[\sum_{n=-\infty}^{\infty} F(n) e^{jn\omega_1 \tau} \left(\frac{1}{T}\right) \int_{-T/2}^{T/2} f(t) e^{jn\omega_1 t} dt\right] .$$

With $\bar{F}(n)$ denoting the complex conjugate of $F(n)$,

$$\phi_{ff}(\tau) = \sum_{n=-\infty}^{\infty} F(n) e^{jn\omega_1 \tau} \bar{F}(n) = \sum_{n=-\infty}^{\infty} F(n) \bar{F}(n) e^{jn\omega_1 \tau} ,$$

$$\Phi_{ff}(\tau) = \sum_{n=-\infty}^{\infty} \left|F(n)\right|^2 e^{jn\omega_1 \tau} .$$

The power spectral density, $\Phi_{ff}(n)$, is defined

$$\Phi_{ff}(n) = \frac{1}{T}\int_{-T/2}^{T/2} \phi_{ff}(\tau) e^{jn\omega_1 \tau} \, d\tau$$

and it can be shown that

$$\Phi_{ff}(n) = \left| F(n) \right|^2 .$$

It can be seen from this relationship that

$$\phi_{ff}(\tau) = \sum_{n=-\infty}^{\infty} \Phi_{ff}(n) e^{jn\omega_1 \tau} .$$

The crosscorrelation function, $\phi_{f_1f_2}(\tau)$, of two periodic signals may be found in a similiar manner if both signals have equal fundamental frequencies, ω_1, and both signals satisfy the Dirichlet conditions. Assuming these conditions are met, then

$$\phi_{f_1f_2}(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} f_1(t) f_2(t+\tau) \, dt .$$

In a fashion similar to that for single signals, it can be shown

$$\phi_{f_1f_2}(\tau) = \sum_{n=-\infty}^{\infty} \overline{F}_1(n) F_2(n) e^{jn\omega_1 \tau} .$$

The cross-power spectral density, $\Phi_{f_1f_2}(n)$, is defined

$$\Phi_{f_1f_2}(n) = \frac{1}{T}\int_{-T/2}^{T/2} \phi_{f_1f_2}(\tau) \, e^{-jn\omega_1 \tau} d\tau$$

and it can be shown that

$$\Phi_{f_1f_2}(n) = \overline{F}_1(n) F_2(n) .$$

Using this relationship

$$\phi_{f_1f_2}(\tau) = \sum_{n=-\infty}^{\infty} \Phi_{f_1f_2}(n) e^{jn\omega_1 \tau} .$$

B. TRANSIENT SIGNALS

A signal, $f(t)$, is defined to be transient if

$$\lim_{t\to\infty} f(t) = 0 .$$

If the transient signal, $f(t)$, satisfies the Dirichlet conditions in any finite interval, and if

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty ,$$

then the signal may be expressed as a Fourier integral [Ref. 5, p. 279]. Under these conditions, the Fourier integral,

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

gives the values of $f(t)$ at all points, including those where the function is not continuous. The Fourier transform of $f(t)$ is

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \quad .$$

The autocorrelation function for the nonperiodic signal is defined

$$\phi_{ff}(\tau) = \int_{-\infty}^{\infty} f(t) f(t+\tau) dt \quad .$$

This can also be written

$$\phi_{ff}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 e^{j\omega\tau} d\omega \quad .$$

Letting

$$\Phi_{ff}(\omega) = |F(j\omega)|^2 \quad ,$$

where $\Phi_{ff}(\omega)$ is defined to be the energy spectral density of the signal $f(t)$, it can be shown that

$$\Phi_{ff}(\omega) = \int_{-\infty}^{\infty} \phi_{ff}(\tau) e^{-j\omega\tau} d\tau \quad ..$$

Thus it can be seen that the energy density spectrum and the autocorrelation function of a transient signal are a Fourier transform pair,

$$\phi_{ff}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{ff}(\omega) e^{j\omega\tau} d\omega$$

and

$$\Phi_{ff}(\omega) = \int_{-\infty}^{\infty} \phi_{ff}(\tau) e^{-j\omega\tau} d\tau \quad ..$$

If two transient signals, $f_1(t)$ and $f_2(t)$, each satisfy the Dirichlet conditions in all finite intervals, and if

$$\int_{-\infty}^{\infty} |f_1(t)| dt < \infty \qquad \text{and} \qquad \int_{-\infty}^{\infty} |f_2(t)| dt < \infty \quad ,$$

then

$$\phi_{f_1 f_2}(\tau) = \int_{-\infty}^{\infty} f_1(t) f_2(t+\tau) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{F}_1(j\omega) F_2(j\omega) e^{j\omega\tau} d\omega \quad ..$$

Now with

$$\Phi_{f_1 f_2}(\omega) = \overline{F}_1(j\omega) F_2(j\omega) \quad ,$$

where $\Phi_{f_1 f_2}(\omega)$ is defined as the cross-energy spectral density of the signals $f_1(t)$ and $f_2(t)$, it can be shown that

$$\Phi_{f_1 f_2}(\omega) = \int_{-\infty}^{\infty} \phi_{f_1 f_2}(\tau) e^{-j\omega\tau} d\tau \quad ,$$

and has as its inverse transform

$$\phi_{f_1 f_2}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{f_1 f_2}(\omega) e^{j\omega\tau} d\omega \quad ..$$

The transient signals $f_1(t)$ and $f_2(t)$ are said to be linearly uncorrelated when $\phi_{f_1 f_2}(\tau) = 0$ for all τ.

C. RANDOM SIGNALS

In general, a random signal, f(t), from a stationary, ergodic random process [Ref. 4, p 279], does not have a Fourier transform since

$$\int_{-\infty}^{\infty} |f(t)|\,dt$$

is not finite. An autocorrelation function may be defined for the random signal f(t) as

$$\phi_{ff}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} f(t)\,f(t+\tau)\,dt \;.$$

Since $\phi_{ff}(\tau)$ satisfies the Dirichlet conditions for all finite intervals and

$$\int_{-\infty}^{\infty} |\phi_{ff}(\tau)|\,d\tau < \infty \;,$$

it can be represented by a Fourier integral. It can then be shown that

$$\phi_{ff}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{ff}(\omega)\,e^{j\omega\tau}\,d\omega \;.$$

where $\Phi_{ff}(\omega)$ is defined as the power spectral density of the signal f(t). $\Phi_{ff}(\omega)$ is the Fourier transform of $\phi_{ff}(\tau)$,

$$\Phi_{ff}(\omega) = \int_{-\infty}^{\infty} \phi_{ff}(\tau)\,e^{-j\omega\tau}\,d\tau \;.$$

If there exist two random signals, $f_1(t)$ and $f_2(t)$, which are sample functions from two different random processes, each of which are stationary, ergodic, and jointly ergodic, then the crosscorrelation function is given as

$$\phi_{f_1 f_2}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} f_1(t)\,f_2(t+\tau)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{f_1 f_2}(\omega)\,e^{j\omega\tau}\,d\omega \,,$$

where $\Phi_{f_1 f_2}(\omega)$ is defined as the cross-power spectral density of the random signals $f_1(t)$ and $f_2(t)$. $\phi_{f_1 f_2}(\tau)$ has the Fourier transform

$$\Phi_{f_1 f_2}(\omega) = \int_{-\infty}^{\infty} \phi_{f_1 f_2}(\tau) e^{-j\omega\tau} d\tau \quad .$$

The random signals $f_1(t)$ and $f_2(t)$ are said to be linearly uncorrelated if $\phi_{f_1 f_2}(\tau) = 0$ for all τ.

III. DESCRIBING FUNCTION AND REMNANT RELATIONS

A. FREQUENCY DOMAIN RELATIONS

Again consider Figure 2 with the input a sample function from an ergodic random process. It is seen that

$$E(j\omega) = I(j\omega) - C(j\omega) \quad ,$$

where the Fourier transforms are as defined in Section B. Now

$$C(j\omega) = \left[N(j\omega) + Y_p(j\omega)E(j\omega)\right] Y_c(j\omega) \quad ,$$

then

$$E(j\omega) = \frac{\left[I(j\omega) - N(j\omega)Y_c(j\omega)\right]}{\left[1 + Y_p(j\omega)Y_c(j\omega)\right]} \quad .$$

Finally, after multiplying by $\overline{I}(j\omega)$, the complex conjugate of the input,

$$\overline{I}(j\omega)E(j\omega) = \frac{\overline{I}(j\omega)\left[I(j\omega) - N(j\omega)Y_c(j\omega)\right]}{\left[1 + Y_p(j\omega)Y_c(j\omega)\right]} \quad .$$

In a similar manner,

$$P(j\omega) = Y_p(j\omega)E(j\omega) + N(j\omega)$$

and then substituting for $E(j\omega)$ from above,

$$P(j\omega) = \frac{\left[Y_p(j\omega)I(j\omega) - N(j\omega)Y_p(j\omega)Y_c(j\omega)\right]}{\left[1 + Y_p(j\omega)Y_c(j\omega)\right]} + N(j\omega)$$

and

$$\overline{I}(j\omega)P(j\omega) = \frac{\overline{I}(j\omega)\left[Y_p(j\omega)I(j\omega) + N(j\omega)\right]}{\left[1 + Y_p(j\omega)Y_c(j\omega)\right]} \quad .$$

Likewise, $\overline{I}(j\omega)C(j\omega)$, $\overline{E}(j\omega)P(j\omega)$, $\overline{E}(j\omega)E(j\omega)$, $\overline{P}(j\omega)P(j\omega)$, and $\overline{C}(j\omega)C(j\omega)$ may be calculated. The results, to be utilized shortly, are shown in Table I.

B. FINITE RUN LENGTH

Since it is impossible to have experimental runs of infinite duration, measurements using finite time histories are necessary. Reference 6 indicates that finite run lengths can be handled analytically as follows: If $i_T(t)$ is the input and defined

$$i_T(t) = \begin{cases} i(t) , & -T \leq t \leq T \\ 0 , & \text{ELSEWHERE} \end{cases}$$

then this function can be considered to be transient and have a Fourier transform

$$I(j\omega) = \int_{-\infty}^{\infty} i_T(t) e^{-j\omega t} dt \; .$$

The other system signals and their transforms can be defined in precisely the same manner. If the run time, T, is large enough to ensure accurate power spectral measurements, yet finite so that the respective Fourier transforms exist, then the following spectral relations are valid [Ref. 6].

$$\Phi_{ii}(\omega) = \lim_{T\to\infty} \left[\frac{1}{2T} \left| I(j\omega) \right|^2 \right]$$

and

$$\Phi_{ip}(\omega) = \lim_{T\to\infty} \left[\frac{1}{2T} \bar{I}(j\omega) P(j\omega) \right]$$

where $\Phi_{ii}(\omega)$ and $\Phi_{ip}(\omega)$ are power and cross-power spectral densities respectively. Utilizing Table I,

$$\Phi_{ip}(\omega) = \lim_{T\to\infty} \left\{ \frac{1}{2T} \left[\frac{\bar{I}(j\omega) I(j\omega) Y_p(j\omega) + \bar{I}(j\omega) N(j\omega) Y_c(j\omega)}{1 + Y_p(j\omega) Y_c(j\omega)} \right] \right\}$$

but

$$\lim_{T\to\infty} \left[\frac{1}{2T} \bar{I}(j\omega) I(j\omega) \right] = \Phi_{ii}(\omega)$$

and

$$\underset{T\to\infty}{\mathrm{LIM}} \left[\frac{1}{2T} \quad \bar{I}(j\omega)N(j\omega) \right] = \Phi_{in}(\omega)$$

thus

$$\Phi_{ip}(\omega) = \frac{Y_p(j\omega)\Phi_{ii}(\omega) + \Phi_{in}(\omega)Y_c(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)} \quad ..$$

Now,

$$\Phi_{in}(\omega) = \int_{-\infty}^{\infty} \phi_{in}(\omega)e^{-j\omega\tau}d\tau \quad ..$$

Since by definition the remnant, n(t), is linearly uncorrelated with the input, i(t), then $\phi_{in}(\tau) = 0$ for all τ. Thus $\Phi_{in}(\omega) = 0$ and

$$\Phi_{ip}(\omega) = \frac{Y_p(j\omega)\Phi_{ii}(\omega)}{1+Y_p(j\omega)Y_c(j\omega)}$$

In a like manner,

$$\Phi_{ie}(\omega) = \underset{T\to\infty}{\mathrm{LIM}} \left[\frac{1}{2T} \bar{I}(j\omega)E(j\omega) \right] \quad ,,$$

or

$$\Phi_{ie}(\omega) = \frac{\Phi_{ii}(\omega) - \Phi_{in}(\omega)Y_c(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)} \quad ..$$

Again since n(t) and i(t) are linearly uncorrelated, $\phi_{in}(\tau) = 0$ for all τ, then $\Phi_{in}(\omega) = 0$. Thus

$$\Phi_{ie}(\omega) = \frac{\Phi_{ii}(\omega)}{1+Y_p(j\omega)Y_c(j\omega)} \quad ..$$

Also

$$\Phi_{pp}(\omega) = \underset{T\to\infty}{\mathrm{LIM}} \left[\frac{1}{2T} \bar{P}(j\omega)P(j\omega) \right]$$

$$\Phi_{pp}(\omega) = \underset{T\to\infty}{\mathrm{LIM}} \left\{ \frac{1}{2T} \left[\frac{\bar{I}(j\omega)\bar{Y}_p(j\omega)+\bar{N}(j\omega)}{1 + \bar{Y}_p(j\omega)\bar{Y}_c(j\omega)} \right] \left[\frac{I(j\omega)Y_p(j\omega)+N(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)} \right] \right\} ,$$

or

$$\Phi_{pp}(\omega) = \frac{\Phi_{ii}(\omega)\left|Y_p(j\omega)\right|^2+\Phi_{ni}(\omega)Y_p(j\omega)+\Phi_{in}(\omega)\overline{Y}_p(j\omega)+\Phi_{nn}(\omega)}{\left|1+Y_p(j\omega)Y_c(j\omega)\right|^2} \quad ..$$

Again, since $\Phi_{in}(\omega) = \Phi_{ni}(\omega) = 0$,

$$\Phi_{pp}(\omega) = \frac{\Phi_{ii}(\omega)\left|Y_p(j\omega)\right|^2+\Phi_{nn}(\omega)}{\left|1+Y_p(j\omega)Y_c(j\omega)\right|^2}$$

C. DESCRIBING FUNCTION AND REMNANT

From $\Phi_{ip}(\omega)$, $\Phi_{ie}(\omega)$, and $\Phi_{pp}(\omega)$, $Y_p(j\omega)$ and $\Phi_{nn}(\omega)$ may be found. Utilizing $\Phi_{ie}(\omega)$ and $\Phi_{ip}(\omega)$,

$$\Phi_{ii}(\omega) = \Phi_{ie}(\omega)\left[1+Y_p(j\omega)Y_c(j\omega)\right]$$

and

$$\Phi_{ii}(\omega) = \frac{1}{Y_p(j\omega)}\Phi_{ip}(\omega)\left[1+Y_p(j\omega)Y_c(j\omega)\right] \quad ..$$

Thus,

$$\Phi_{ie}(\omega)\left[1+Y_p(j\omega)Y_c(j\omega)\right] = \frac{1}{Y_p(j\omega)}\Phi_{ip}(\omega)\left[1+Y_p(j\omega)Y_c(j\omega)\right] \quad ,.$$

or

$$Y_p(j\omega) = \frac{\Phi_{ip}(\omega)}{\Phi_{ie}(\omega)} \quad ..$$

Also, from $\Phi_{pp}(\omega)$,

$$\Phi_{nn}(\omega) = \left|1+Y_p(j\omega)Y_c(j\omega)\right|^2\Phi_{pp}(\omega)-\left|Y_p(j\omega)\right|^2\Phi_{ii}(\omega) \quad ..$$

In addition, $Y_c(j\omega)$ can be determined and calculated from

$$\Phi_{ic}(\omega) = \Phi_{in}(\omega)Y_c(j\omega) + \Phi_{ie}(\omega)Y_p(j\omega)Y_c(j\omega) \quad ..$$

Again $\Phi_{in}(\omega) = 0$, thus

$$\Phi_{ic}(\omega) = \Phi_{ie}(\omega) Y_p(j\omega) Y_c(j\omega) \quad ..$$

But

$$\Phi_{ie}(\omega) Y_p(j\omega) = \Phi_{ip}(\omega) \quad ,$$

thus

$$Y_c(j\omega) = \frac{\Phi_{ic}(\omega)}{\Phi_{ip}(\omega)} \quad ..$$

The functions in Table II form the basis of the describing function measurement technique utilized in this study.

D. SINUSOIDAL INPUTS

The relations in Table II are predicated on the existence of a random input. In experimental work, a random appearing input is often used and can be generated as a summation of sine waves [Ref. 3]. The input can thus be represented by

$$i(t) = \sum_{k=1}^{n} A_k \sin\omega_k t$$

where the ω_k are chosen such that in a finite run length there will exist an integral number of periods or complete cycles. In this analysis a run time of 150 seconds was used and $0.08 \leq \omega_k \leq 40.0$ radians per second.

Utilization of a sinusoidal input results in system signals that have both random and periodic components. With what is now a mixed signal, the question arises as to which power spectral relationship should be used. The solution is to use the periodic power spectral relationships for measurements made at the input frequencies and to use the random, finite relationships at all other frequencies.

It should be noted that if the experimental run length, T, is large and contains an integral number of periods of each of the input sinusoids, then the Fourier transforms of the periodic and finite random signals differ only by a constant of proportionality [Ref. 6].

In the periodic case, it was shown that

$$F_k(n) = \frac{1}{2T_k}\int_{-T_k}^{T_k} f(t)\,e^{-jn\omega_1 t}dt ,$$

and for the random signal,

$$F(j\omega) = \int_{-\infty}^{\infty} f_T(t)\,e^{-j\omega t}dt .$$

If it is recalled that $f_T(t) = 0$ for $t<-T$ and $t>T$, and letting $T = m_k T_k$, where m_k is the number of periods, T_k, of frequency ω_k, then

$$F_k(n) = \frac{1}{2T_k}\int_{-T_k}^{T_k} f(t)\,e^{-jn\omega_1 t}dt = \frac{1}{2m_k T_k}\int_{-m_k T_k}^{m_k T_k} f(t)\,e^{-jn\omega_1 t}dt ,$$

or

$$F_k(n) = \frac{1}{2T}\int_{-T}^{T} f(t)\,e^{-j\omega_k t}dt ;$$

thus it can be seen that

$$2TF_k(n) = F(j\omega_k) .$$

It should be noted that the same expansion of the limits on the integral can be used for the periodic function; thus

$$\Phi_{ff}(n_k) = \frac{1}{2T}\int_{-T}^{T} \frac{1}{2T}\int_{-T}^{T} f(t)\,f(t+\tau)\,dt\,e^{-jn\omega_k \tau}d\tau ,$$

or

$$\Phi_{ff}(n_k) = \frac{1}{4T^2}\int_{-T}^{T}\int_{-T}^{T} f(t)\,f(t+\tau)\,e^{-jn\omega_k\tau}\,dt\,d\tau \quad .$$

Also for the finite random function,

$$\Phi_{ff}(\omega)_T = \int_{-T}^{T} \phi_{ff}(\tau)_T\,e^{-j\omega\tau}\,d\tau \quad ,$$

or

$$\Phi_{ff}(\omega)_T = \frac{1}{2T}\int_{-T}^{T}\int_{-T}^{T} f_T(t)\,f_T(t+\tau)\,e^{-j\omega\tau}\,dt\,d\tau \quad ;$$

then by equating the integrals, since $f_T(t) = f(t)$ for $-T\leq t\leq T$, it is seen that at a specific input frequency, ω_k,

$$\Phi_{ff}(\omega_k)_T = 2T\Phi_{ff}(n_k) \quad ,$$

where

$$\Phi_{ff}(\omega_k)_T = \underset{T\to\infty}{\mathrm{LIM}}\left[\frac{1}{2T}\left|F(j\omega_k)\right|^2\right] \quad .$$

It has been shown that

$$Y_p(j\omega) = \frac{\Phi_{ip}(\omega)}{\Phi_{ie}(\omega)} \quad .$$

At the input frequencies, with T large and containing an integral number of periods of each input frequency,

$$Y_p(j\omega_k) = \frac{\Phi_{ip}(\omega_k)}{\Phi_{ie}(\omega_k)} \doteq \frac{\Phi_{ip}(\omega_k)_T}{\Phi_{ie}(\omega_k)_T} = \frac{\Phi_{ip}(n_k)\,2T}{\Phi_{ie}(n_k)\,2T} \quad .$$

or

$$Y_p(j\omega_k) \doteq \frac{\Phi_{ip}(n_k)}{\Phi_{ie}(n_k)} = \frac{\overline{I}(n_k)\,P(n_k)}{\overline{I}(n_k)\,E(n_k)} = \frac{P(n_k)}{E(n_k)} \quad .$$

Similarly,

$$Y_c(j\omega_k) \doteq \frac{\Phi_{ic}(n_k)}{\Phi_{ip}(n_k)} = \frac{C(n_k)}{P(n_k)} \quad .$$

This illustrates that the cross-power spectral measurements need not be made and only the Fourier transforms are needed. The latter is usually an easier measurement than the former.

The terms in $\Phi_{nn}(\omega)$ can be examined in a similar manner. In this case, if measurements are taken at frequencies, ω_h, other than those used in the input, then the expression for $\Phi_{nn}(\omega)$ is somewhat simplified; i.e.,

$$\Phi_{nn}(\omega_h) = \left|1+Y_p(j\omega_h)Y_c(j\omega_h)\right|^2 \Phi_{pp}(\omega_h) \quad ,$$

since $\Phi_{ii}(\omega_h) = 0$.

It should be emphasized that at frequencies, ω_h, other than those used in the input,

$$Y_p(j\omega_h) \neq \frac{P(n_h)}{E(n_h)} \qquad \text{and} \qquad Y_c(j\omega_h) \neq \frac{C(n_h)}{P(n_h)} \quad ,$$

since $I(n_h) = \bar{I}(n_h) = 0$. Thus $Y_p(j\omega_h)$ and $Y_c(j\omega_h)$ must be estimated, as direct calculations can be made only at the input frequencies. In order to estimate $Y_p(j\omega_h)$, simple linear interpolation between $Y_p(j\omega_k)$ and $Y_p(j\omega_{k+1})$ can be used.

IV. COMPUTER MECHANIZATION

A. EXPERIMENTAL SET-UP

The measurement of a human's describing function and remnant in the compensatory tracking task of Figure 2 was made using a hybrid (analog-digital) computer. The error signal, e(t), was viewed as the vertical displacement of a horizontal line on an oscilloscope screen. The operator's controller consisted of a non-moving force stick. Control was effected by fore and aft pressure on the stick; e.g., if the line on the oscilloscope moved above the datum, the operator applied forward pressure to move the line down, and vice-versa. The input, i(t), and controlled element dynamics, $Y_c(j\omega)$, were mechanized on the computer as were the measurement algorithms to be described. Each experimental tracking run lasted 150 seconds.

B. FAST FOURIER TRANSFORM

The pertinent relationships are again

$$Y_p(j\omega_k) = \frac{P(n_k)}{E(n_k)}$$

and

$$\Phi_{nn}(\omega_h) = \left|1+Y_p(j\omega_h)Y_c(j\omega_h)\right|^2 \Phi_{pp}(\omega_h) \quad .$$

Now

$$P(n_k) = \frac{1}{T}\int_{-T/2}^{T/2} p(t)e^{-j\omega_k t}dt \qquad ,$$

or

$$P(n_k) = \frac{1}{T}\left[\int_{-T/2}^{T/2} p(t)\cos(n\omega_k t)\,dt + j\int_{-T/2}^{T/2} p(t)\sin(n\omega_k t)\,dt\right] \quad ..$$

These integrals may be approximated by the following summations:

$$P(n_k) \doteq \frac{\Delta t}{T}\sum_{n=0}^{N} p(n\Delta t)\cos(\omega_k n\Delta t) + j\,\frac{\Delta t}{T}\sum_{n=0}^{N} p(n\Delta t)\sin(\omega_k n\Delta t) \quad ..$$

If

$$A_{p_k} = \sum_{n=0}^{N} p(n\Delta t)\ \cos(\omega_k n\Delta t)$$

and

$$B_{p_k} = \sum_{n=0}^{N} p(n\Delta t)\ \sin(\omega_k n\Delta t) \quad ,$$

then the fast Fourier transform $P(n_k)$ can be written as

$$P(n_k) \doteq \frac{\Delta t}{T}\left[A_{p_k} + jB_{p_k}\right] \quad ..$$

Similarly, it can be shown that

$$C(n_k) \doteq \frac{\Delta t}{T}\left[A_{c_k} + jB_{c_k}\right] \qquad \text{and} \qquad E(n_k) \doteq \frac{\Delta t}{T}\left[A_{e_k} + jB_{e_k}\right] \quad ..$$

From this,

$$Y_p(j\omega_k) \doteq \frac{A_{p_k} + jB_{p_k}}{A_{e_k} + jB_{e_k}}$$

and

$$\left|Y_p(j\omega_k)\right| = \left[\frac{A_{p_k}^{\ 2} + B_{p_k}^{\ 2}}{A_{e_k}^{\ 2} + B_{e_k}^{\ 2}}\right]^{1/2} \quad ..$$

It can also be shown that

$$\angle Y_p(j\omega_k) = \tan^{-1}\frac{B_{e_k}}{A_{e_k}} - \tan^{-1}\frac{B_{p_k}}{A_{p_k}} \quad ..$$

In order to validate the simulated controlled element dynamics, $Y_c(j\omega)$, on-line measurement of $Y_c(j\omega)$ can be utilized:

$$|Y_c(j\omega_k)| = \left[\frac{A_{c_k}^2 + B_{c_k}^2}{A_{p_k}^2 + B_{p_k}^2}\right]^{1/2}$$

and

$$\measuredangle Y_c(j\omega_k) = \tan^{-1}\frac{B_{p_k}}{A_{p_k}} - \tan^{-1}\frac{B_{c_k}}{A_{c_k}} .$$

The power spectra of the remnant can also be determined from the above relationships with the interpolation process described earlier. The determination of the power spectra of the operator output, $\Phi_{pp}(\omega_h)$, can be accomplished using the measurements p(t) at any desired frequency [Ref. 6]. Previously it was shown that

$$\Phi_{pp}(\omega_h) \doteq \Phi_{pp}(n_h)_T = \Phi_{pp}(n_h)2T ,$$

thus

$$\Phi_{pp}(\omega_h) \doteq \bar{P}(n_h)P(n_h)2T = 2T|P(n_h)|^2 = \frac{(\Delta t)^2}{T^2}2T|A_{p_h}^2 + B_{p_h}^2| ,$$

and finally,

$$\Phi_{pp}(\omega_h) \doteq \frac{2(\Delta t)^2}{T}|A_{p_h}^2 + B_{p_h}^2| .$$

From this it can be seen that in order to determine the operator's describing function, $Y_p(j\omega_k)$, the remnant, $\Phi_{nn}(\omega_h)$, and the controlled element dynamics, $Y_c(j\omega_k)$, the only measurements needed are those of the error, operator output, and controlled element output. If each of those measurements, taken at specific times, is then multiplied

by the proper trigonometric function and summed over the entire run, then the describing functions and remnant can be calculated.

The major drawback of the fast Fourier transform technique is the necessity of calling the trigonometric functions sin(x) and cos(x) during the run. This requires so much computer time that the multiplication and addition computations cannot be performed between analog-to-digital interrupts. This in turn means that all data must be stored for later computational purposes.

The necessity of calling trigonometric functions during the run can be avoided if the relationships for sin(a+b) and cos(a+b) are utilized in recursive fashion. If the initial time of the run is "a" and the time between measurements is "b" then

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

and

$$\cos(a+b) = \cos(a)\cos(b) + \sin(a)\sin(b).$$

It is obvious that a recursive process can be mechanized which obviates calling trigonometric functions during the run.

The use of this method yields results immediately upon completion of the run and conserves computer storage space. Within seconds of run completion, the data may be analyzed from either numerical or graphical read-out.

V. RESULTS

The operation of the program was checked by measuring the transfer function of known elements or filters in place of the operator. The results are shown in Figure 3, 4, 5, and 6.

In validating the remnant measurement technique, the operator was again replaced by an element with known transfer function. A signal

$$n(t) = A\sin(\omega_h t), \quad \omega_k < \omega_h < \omega_{k+1}$$

was inserted at the output of this element. Measured values of $\Phi_{nn}(\omega)$ were then compared with the theoretical $\frac{A^2}{4}$ value [Ref. 7].

Actual describing function and remnant measurements were then taken on two subjects. The first of these had considerable previous experience. The second subject had experience only with this experimental set-up. The measurement results are shown in Figures 7-18 for controlled elements of $Y_c(s) = 1.0$ and $\frac{1}{s}$. Each of these figures represents the results of 10 tracking runs. In all runs, the human describing function contains the gain of the controller.

The describing functions illustrated are comparable with those obtained by other experimenters; e.g., [Ref. 8].

This computer program represents a powerful tool for use in experimental investigations involving a human

controller. It can, for example, be utilized in a variety of situations in which quantitative models of pilot behavior are desired. The program itself requires little core storage. This means that considerable storage is available for simulating complex aircraft dynamics, providing detailed display formats and calculating performance measures.

TABLE I

$$E(j\omega) = \frac{I(j\omega) - N(j\omega)Y_c(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)}$$

$$P(j\omega) = \frac{I(j\omega)Y_p(j\omega) + N(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)}$$

$$C(j\omega) = N(j\omega)Y_c(j\omega) + E(j\omega)Y_p(j\omega)Y_c(j\omega)$$

$$\bar{I}(j\omega)E(j\omega) = \frac{\bar{I}(j\omega)I(j\omega) - \bar{I}(j\omega)N(j\omega)Y_c(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)}$$

$$\bar{I}(j\omega)P(j\omega) = \frac{\bar{I}(j\omega)I(j\omega)Y_p(j\omega) + \bar{I}(j\omega)N(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)}$$

$$\bar{I}(j\omega)C(j\omega) = \left[\bar{I}(j\omega)N(j\omega) + \bar{I}(j\omega)E(j\omega)Y_p(j\omega)\right] Y_c(j\omega)$$

$$\bar{E}(j\omega)E(j\omega) = \left[\frac{\bar{I}(j\omega) - \bar{N}(j\omega)\bar{Y}_c(j\omega)}{1 + \bar{Y}_p(j\omega)\bar{Y}_c(j\omega)}\right]\left[\frac{I(j\omega) - N(j\omega)Y_c(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)}\right]$$

$$\bar{P}(j\omega)P(j\omega) = \left[\frac{\bar{I}(j\omega)\bar{Y}_p(j\omega) + \bar{N}(j\omega)}{1 + \bar{Y}_p(j\omega)\bar{Y}_c(j\omega)}\right]\left[\frac{I(j\omega)Y_p(j\omega) + N(j\omega)}{1 + Y_p(j\omega)Y_c(j\omega)}\right]$$

$$\bar{C}(j\omega)C(j\omega) = \left[\bar{N}(j\omega) + \bar{E}(j\omega)\bar{Y}_p(j\omega)\right]\left[\bar{Y}_c(j\omega)\right]\left[N(j\omega) + E(j\omega)Y_p(j\omega)\right]\left[Y_c(j\omega)\right]$$

TABLE II

$$Y_p(j\omega) = \frac{\Phi_{ip}(\omega)}{\Phi_{ie}(\omega)}$$

$$Y_c(j\omega) = \frac{\Phi_{ic}(\omega)}{\Phi_{ip}(\omega)}$$

$$\Phi_{nn}(\omega) = \Phi_{pp}(\omega)|1 + Y_p(j\omega)Y_c(j\omega)|^2 - \Phi_{ii}(\omega)|Y_p(j\omega)|^2$$

$$\Phi_{pp}(\omega) = \frac{\Phi_{ii}(\omega)|Y_p(j\omega)|^2 + \Phi_{nn}(\omega)}{|1 + Y_p(j\omega)Y_c(j\omega)|^2}$$

HUMAN OPERATOR in a COMPENSATORY TRACKING TASK

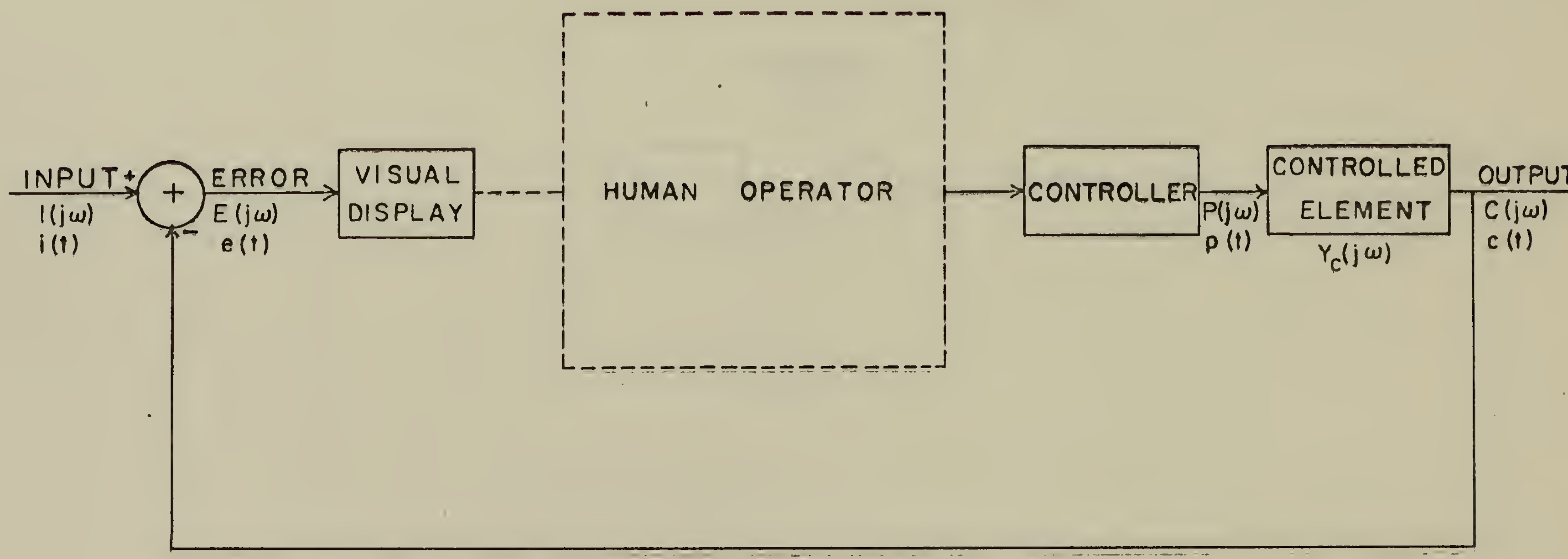


FIGURE 1

QUASI-LINEAR MODEL of HUMAN OPERATOR in a COMPENSATORY TRACKING TASK

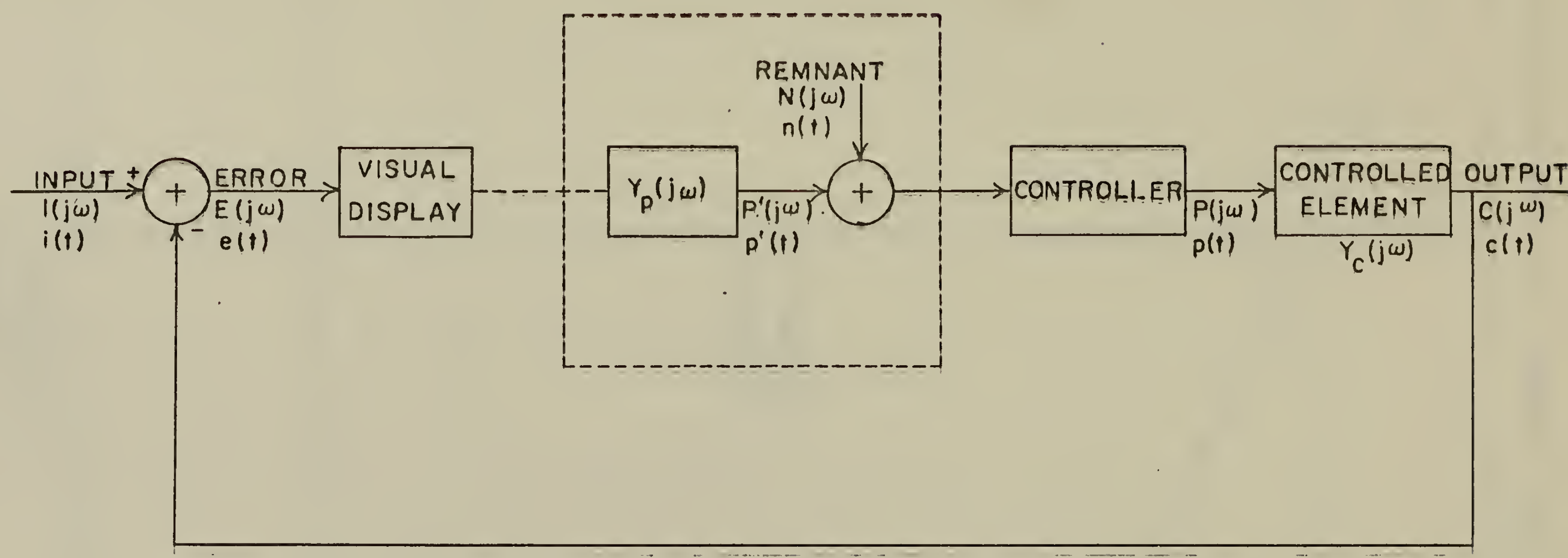


FIGURE 2

MEASUREMENT of KNOWN $(\frac{1}{S})$ ELEMENT

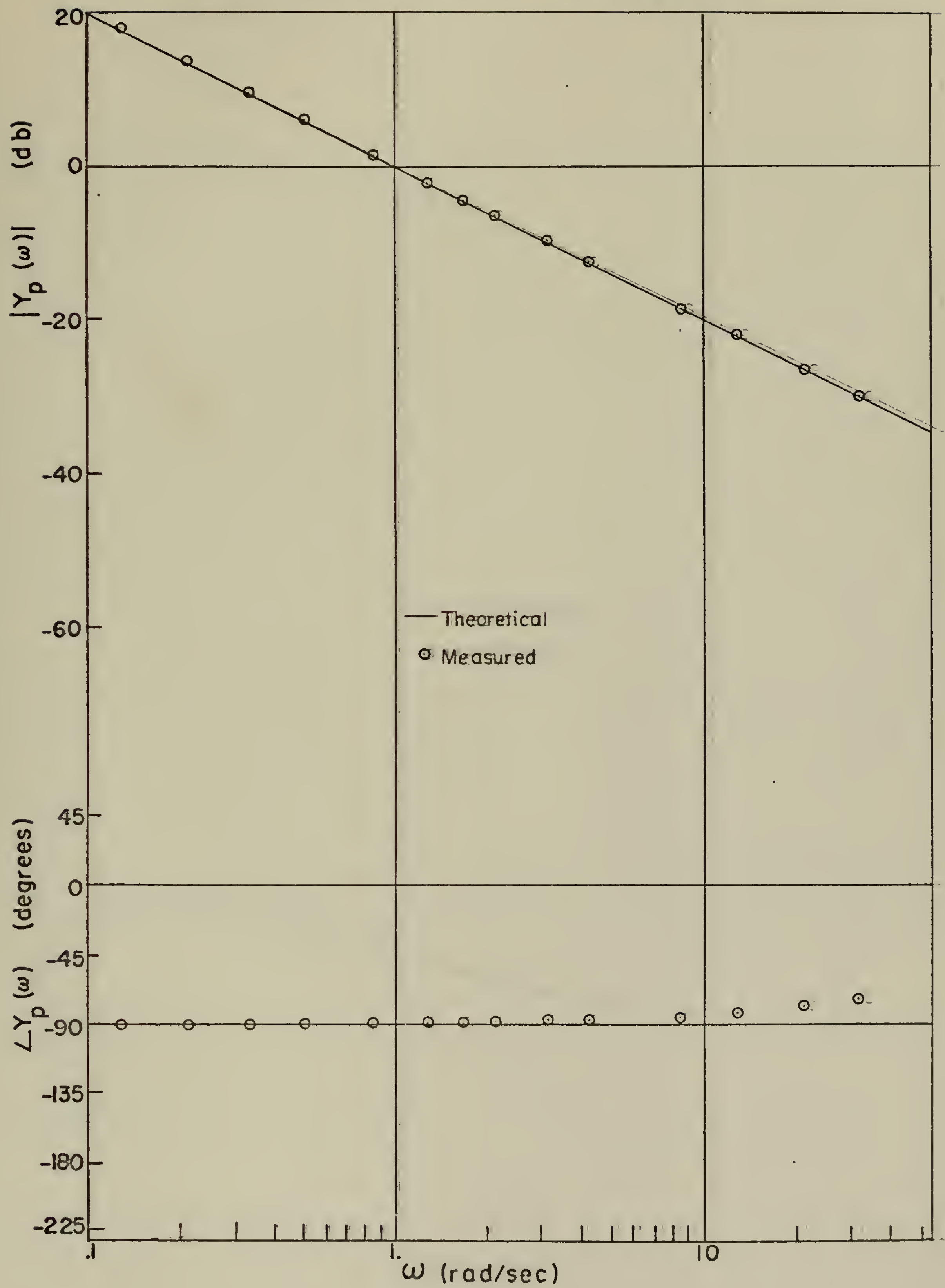


FIGURE 3

MEASUREMENT of KNOWN $\left(\frac{1}{S+1}\right)$ ELEMENT

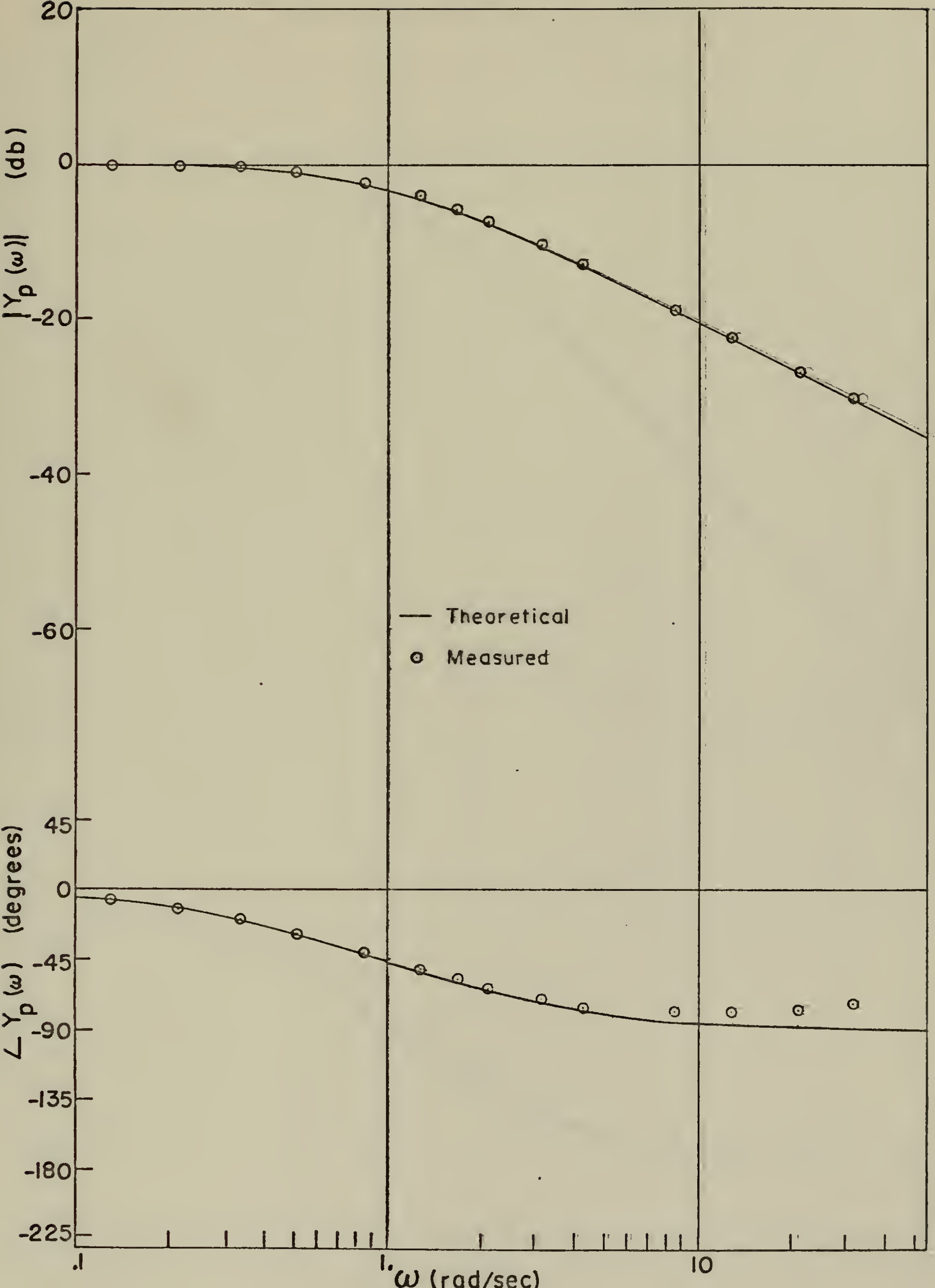


FIGURE 4

MEASUREMENT of KNOWN $\left(\frac{1}{(S+1)^2}\right)$ ELEMENT

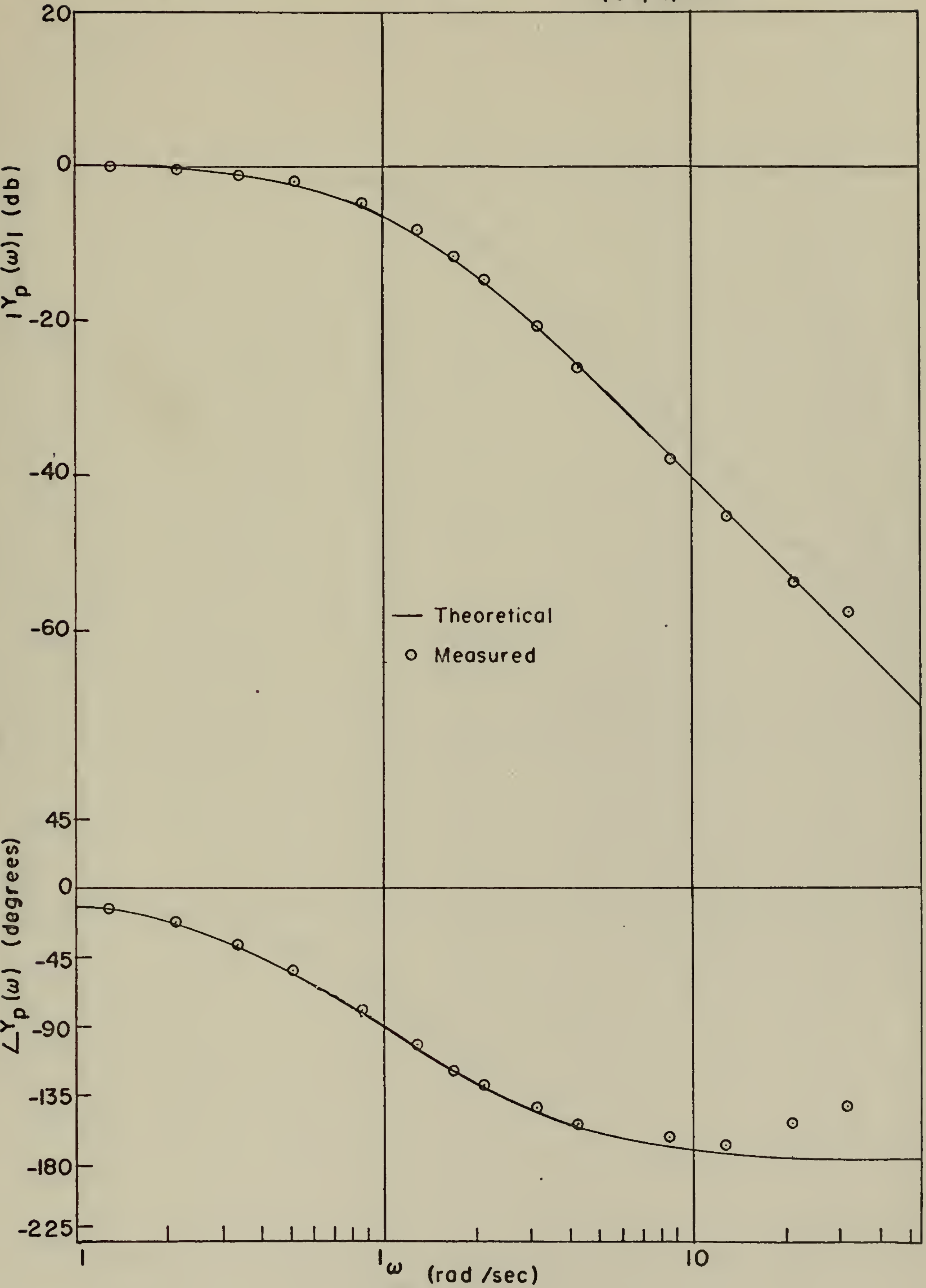
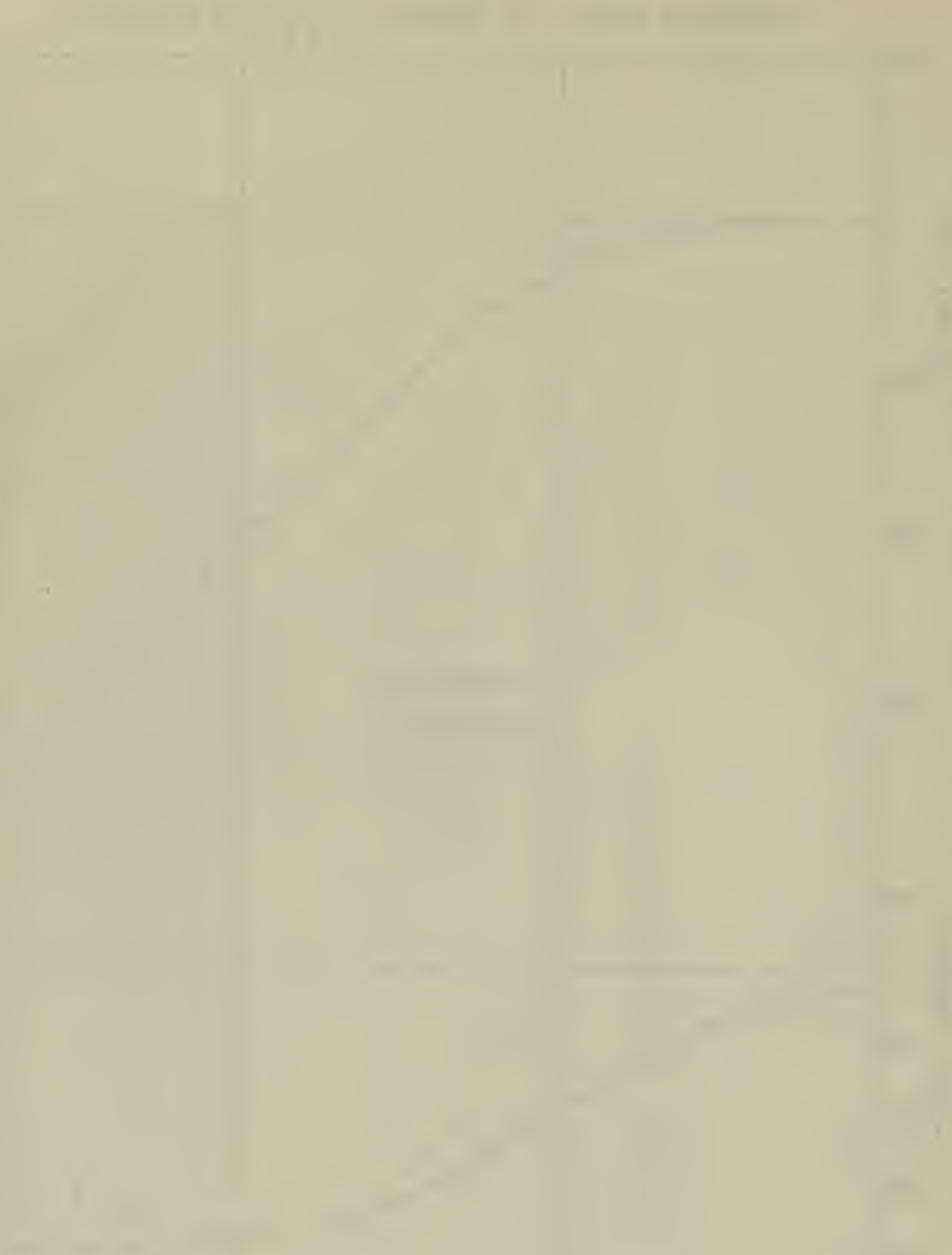
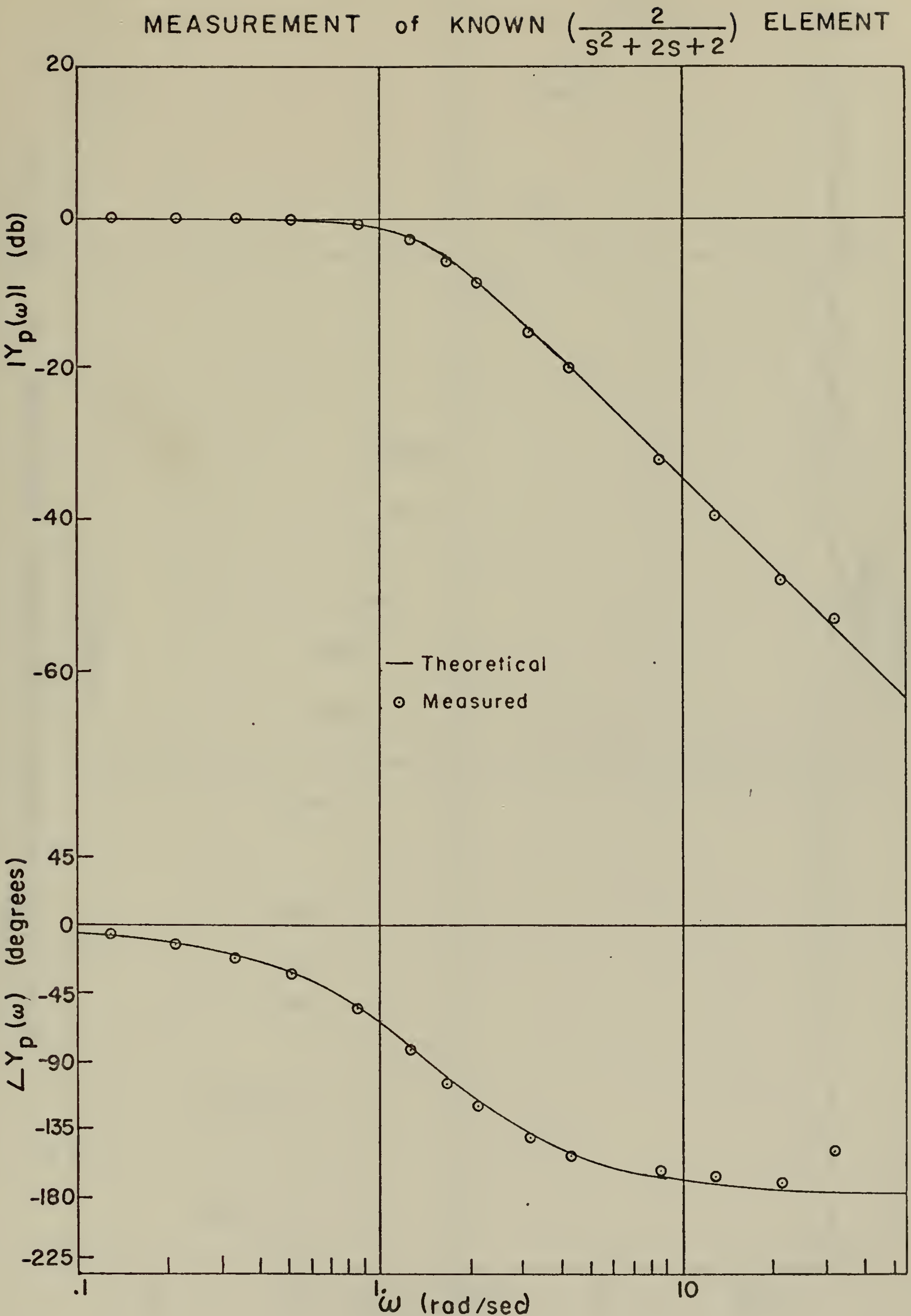


FIGURE 5





MEASUREMENT of KNOWN $\left(\frac{2}{s^2+2s+2}\right)$ ELEMENT
20
0
-20
-40
-60
$|Y_p(\omega)|$ (db)
45
0
-45
-90
-135
-180
-225
$\angle Y_p(\omega)$ (degrees)
— Theoretical
⊙ Measured
.1
1
10
ω (rad/sec)

FIGURE 6

AMPLITUDE of HUMAN DESCRIBING FUNCTION for $Y_c(s) = 1.0$

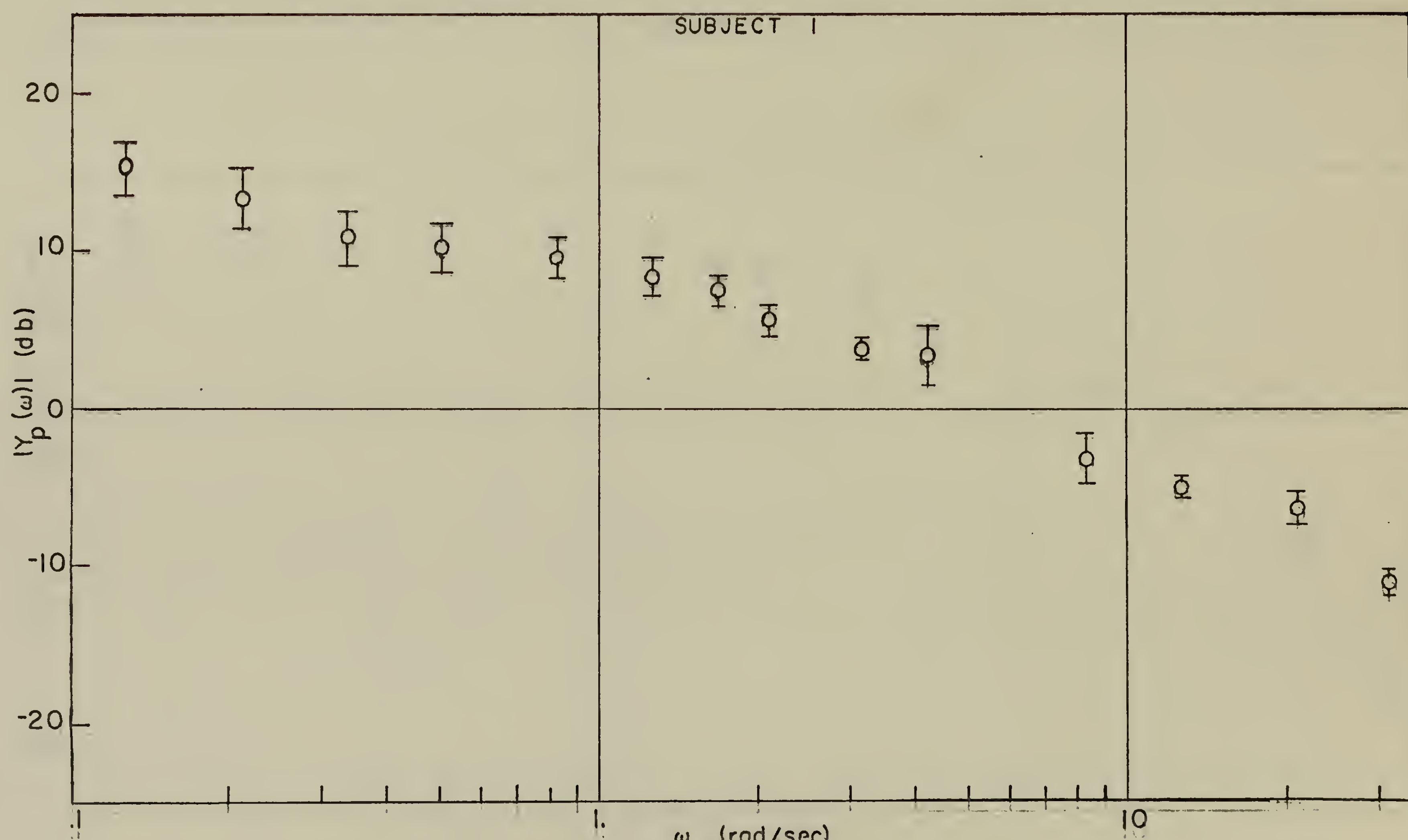


FIGURE 7

PHASE of HUMAN DESCRIBING FUNCTION for $Y_c(s) = 1.0$

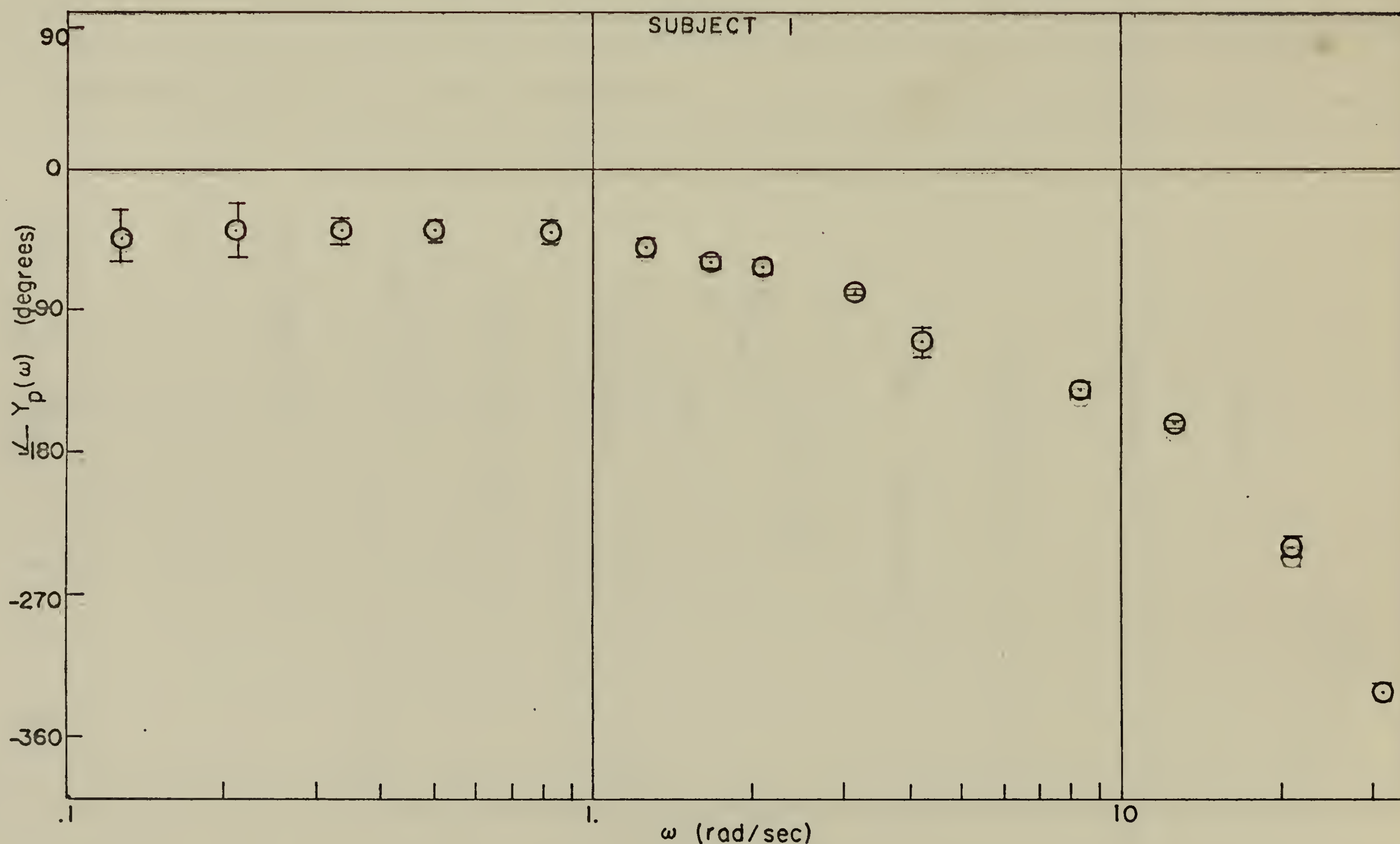


FIGURE 8

REMNANT of HUMAN DESCRIBING FUNCTION for $Y_c(s) = 1.0$

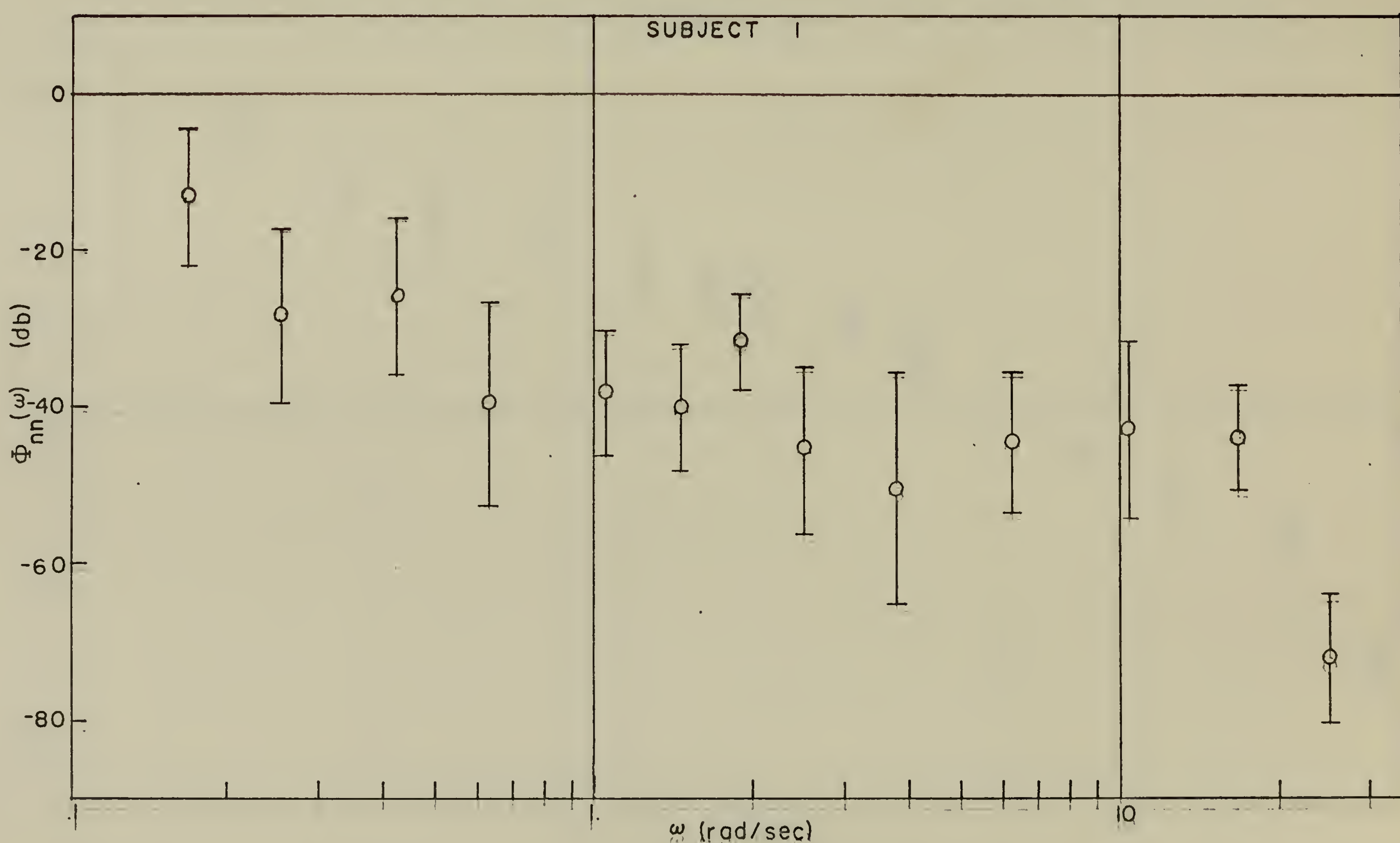


FIGURE 9

AMPLITUDE of HUMAN DESCRIBING FUNCTION for $Y_c(s) = 1.0$

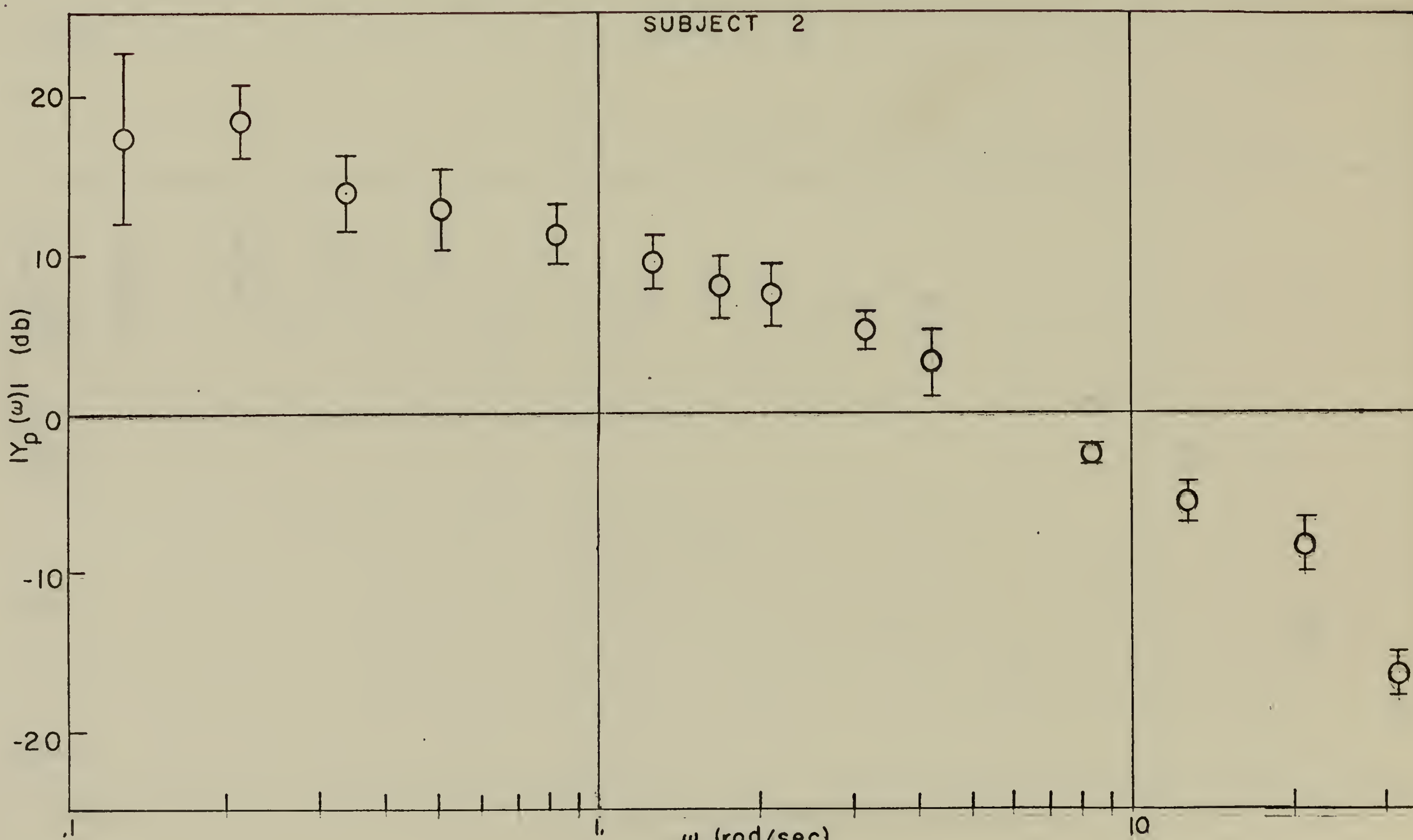


FIGURE 10

PHASE of HUMAN DESCRIBING FUNCTION for $Y_c(s) = 1.0$

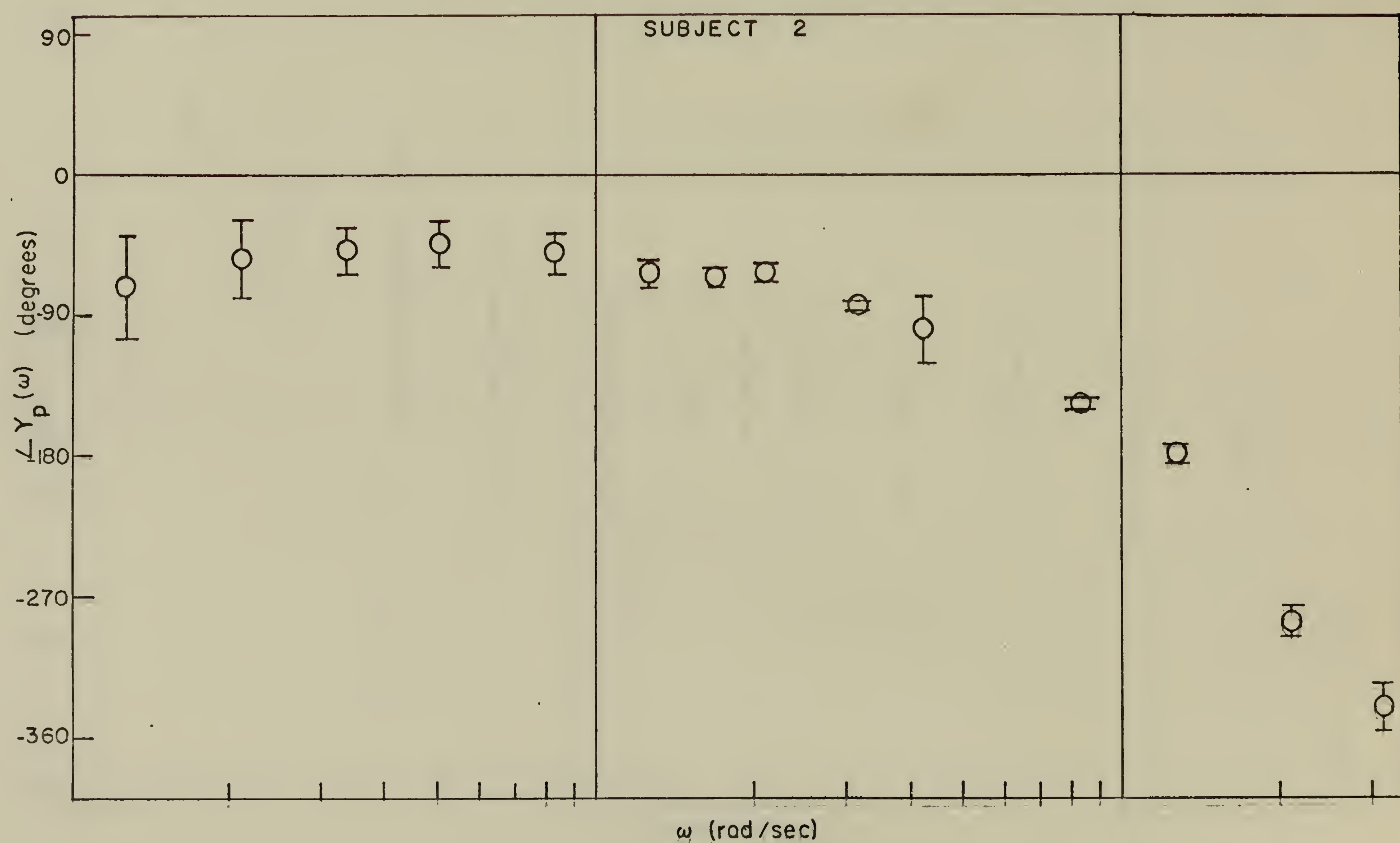


FIGURE 11

REMNANT of HUMAN DESCRIBING FUNCTION for $Y_c(s) = 1.0$

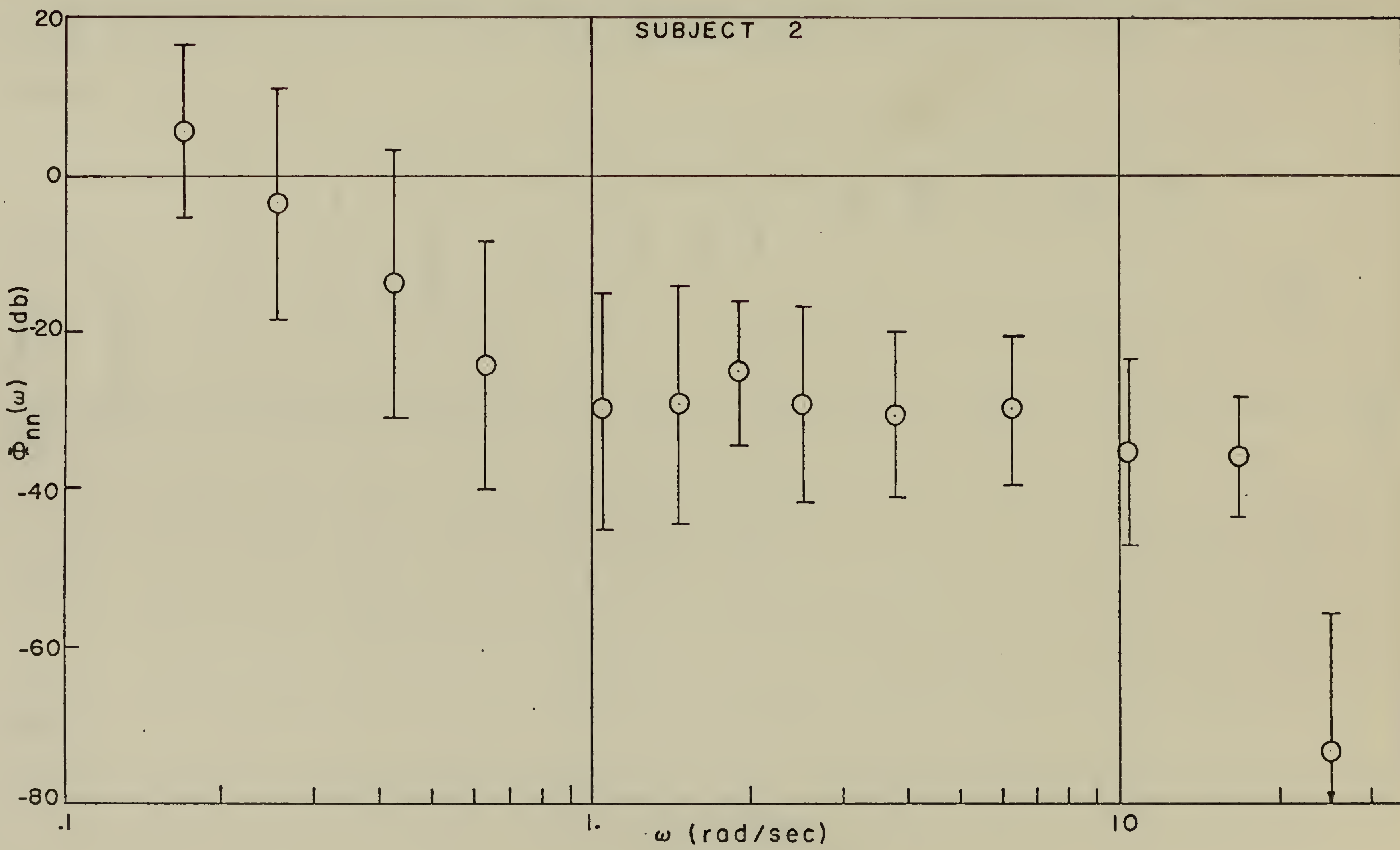


FIGURE 12

AMPLITUDE of HUMAN DESCRIBING FUNCTION for $Y_c(s) = \frac{1}{s}$

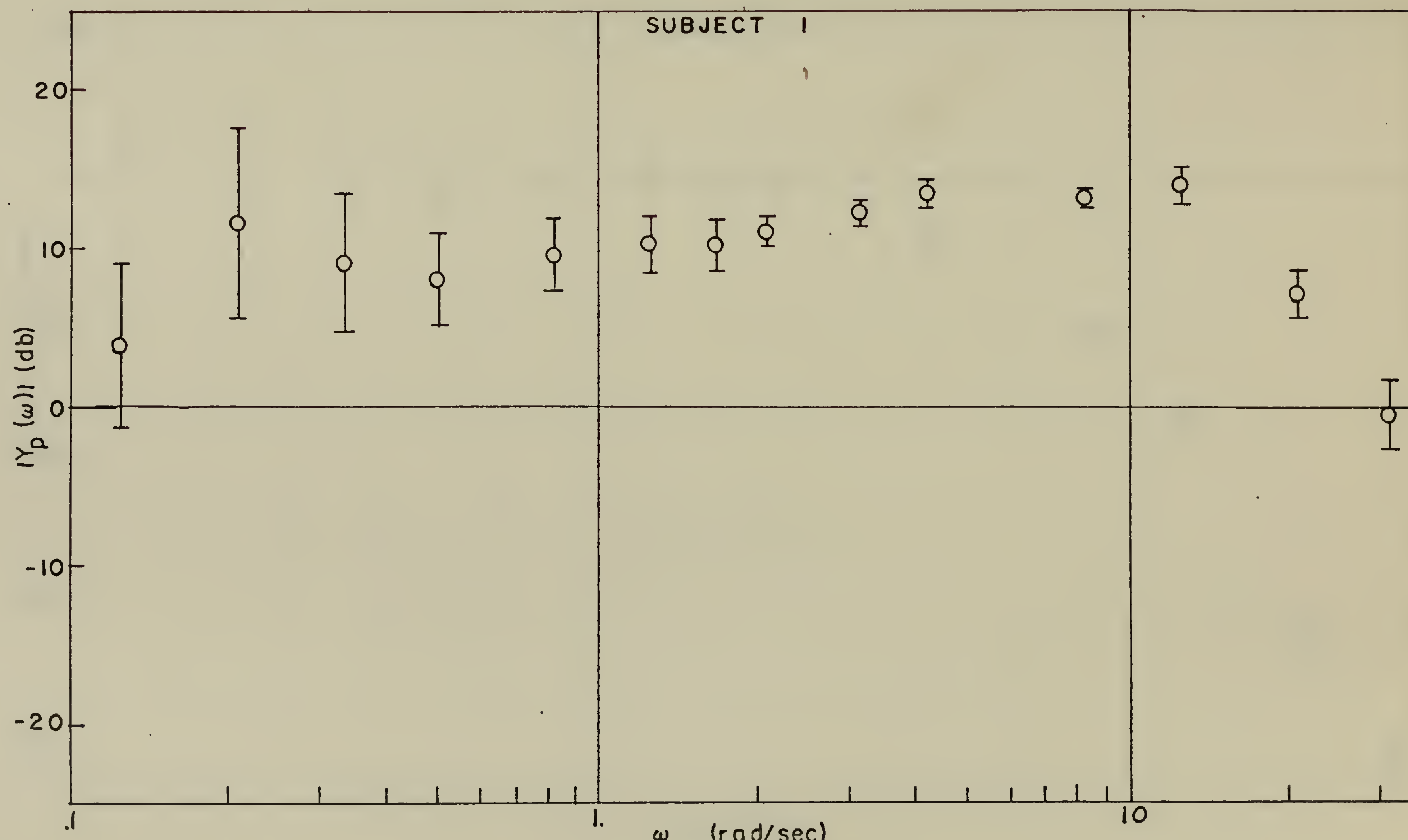


FIGURE 13

PHASE of HUMAN DESCRIBING FUNCTION for $Y_C(s) = \frac{1}{s}$

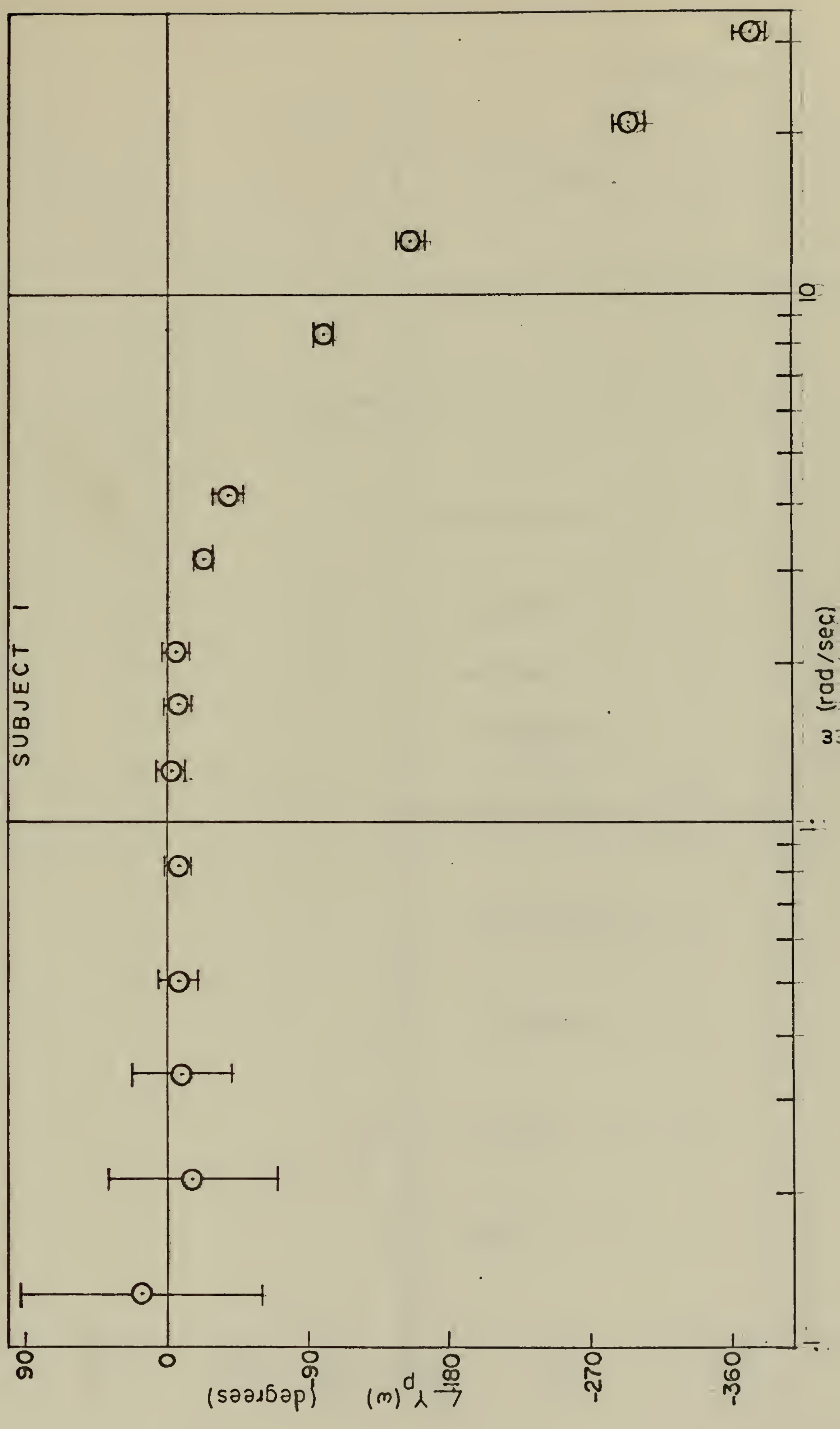


FIGURE 14

REMNANT of HUMAN DESCRIBING FUNCTION for $Y_c(s) = \frac{1}{s}$

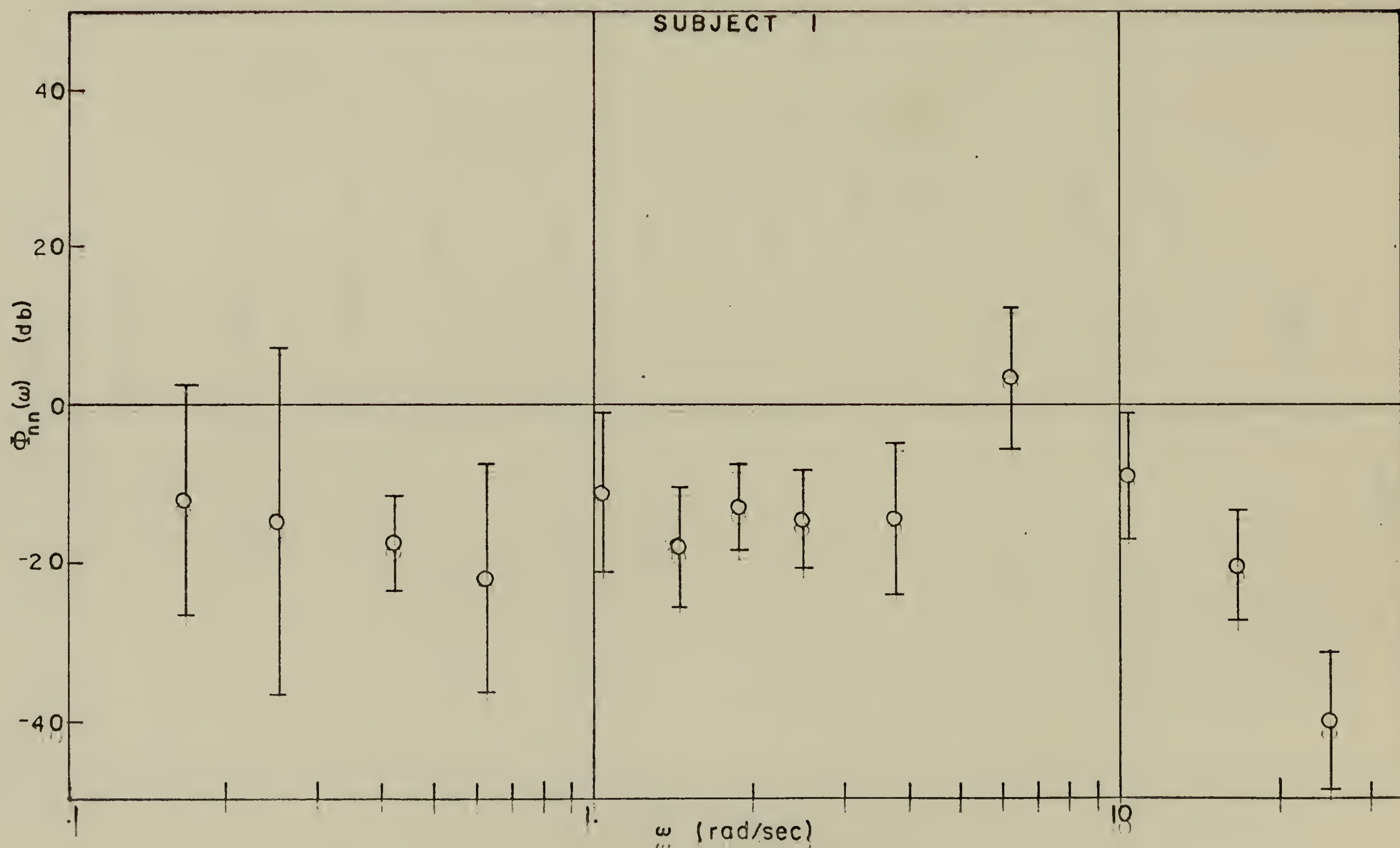


FIGURE 15

AMPLITUDE of HUMAN DESCRIBING FUNCTION for $Y_c(s) = \frac{1}{s}$

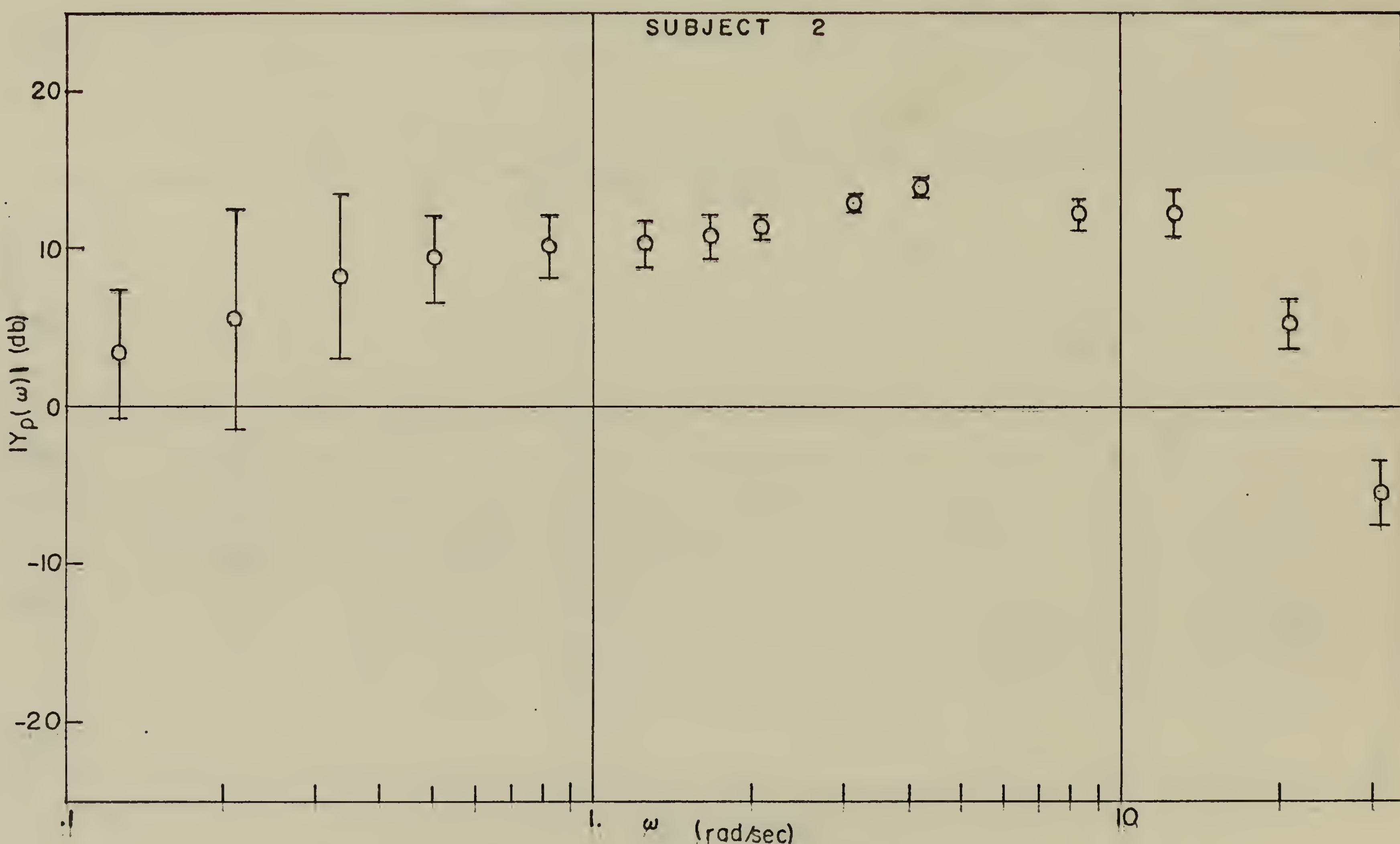


FIGURE 16

PHASE of HUMAN DESCRIBING FUNCTION for $Y_c(s) = \frac{1}{s}$

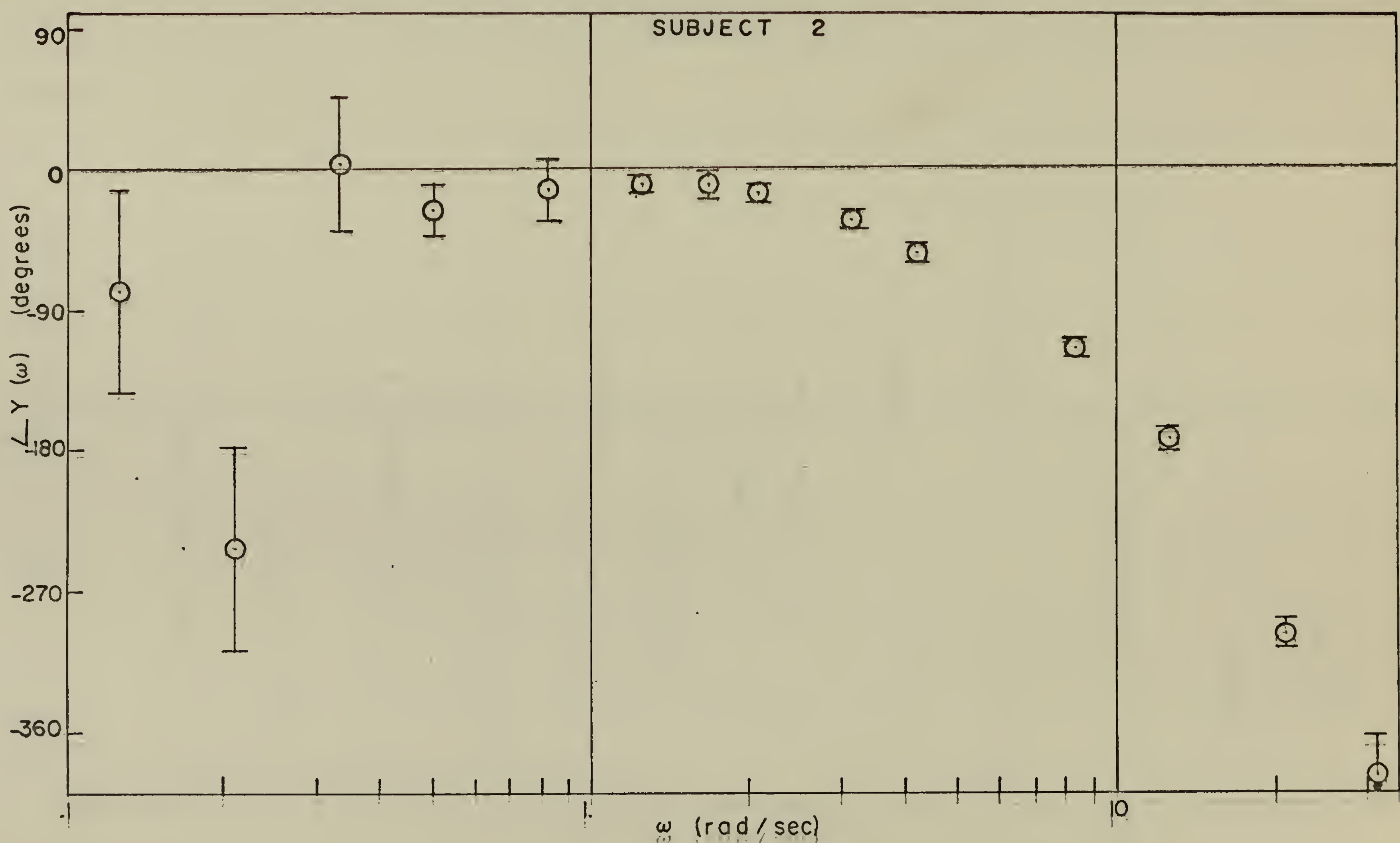


FIGURE 17

REMNANT of HUMAN DESCRIBING FUNCTION for $Y_c(s) = \frac{1}{s}$

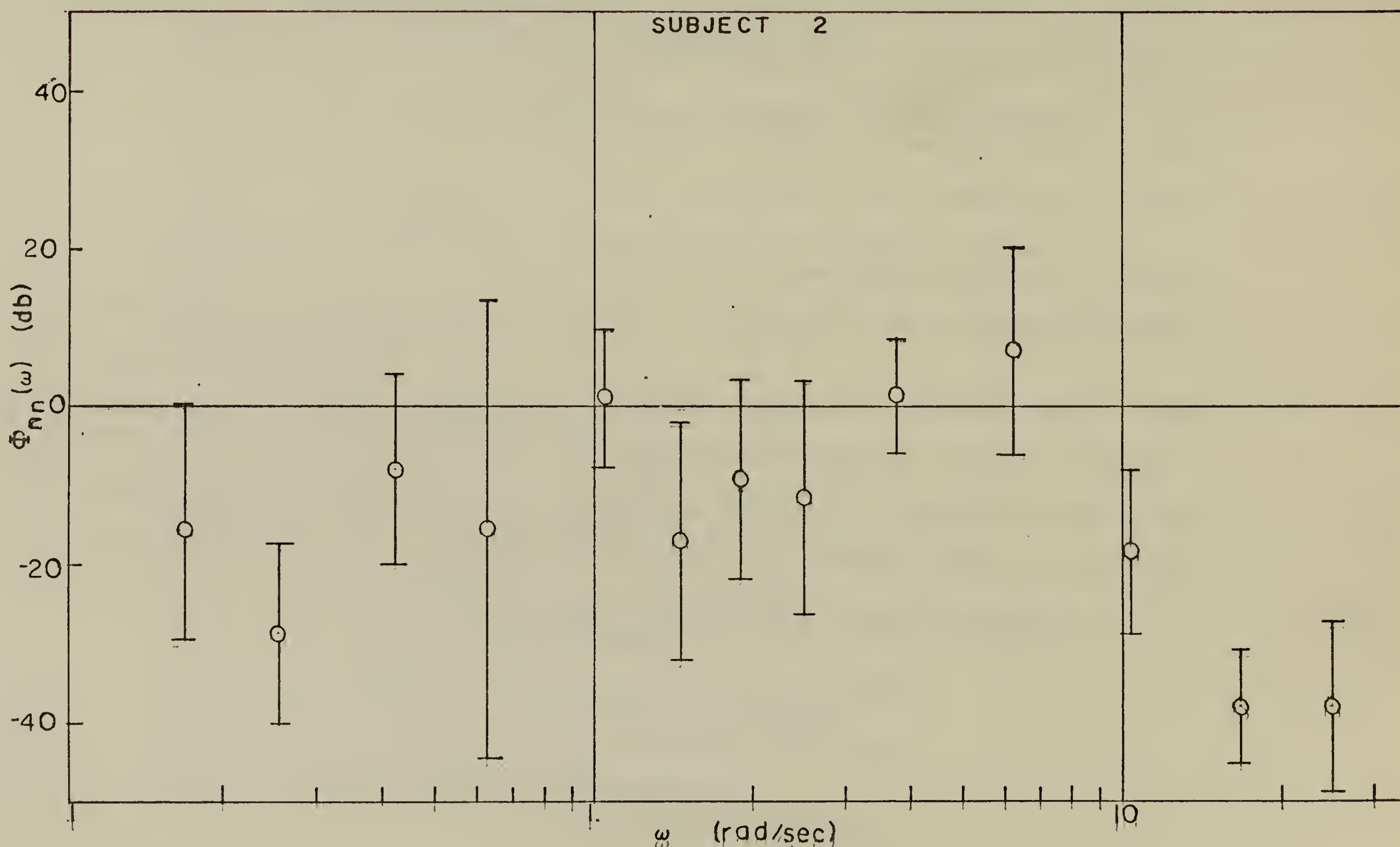


FIGURE 18

```
100 FORMAT (9X,1F10.5,1F15.7)
101 FORMAT (1H1,8X,1HK,10X,6HAMP(K),12X,4HW(K),//(7X,I4,6X,1F10.5,4X,1
   1F15.7))
102 FORMAT (1H1,4X,1HK,10X,5HAE(K),10X,5HBF(K),10X,5HA1(K),10X,5HB1(K)
   1,10X,5HA2(K),10X,5HB2(K),//,(I5,2X,6F15.7))
103 FORMAT (///,4X,1HK,11X,4HW(K),10X,5HYP(K),8X,7HPHAP(K),10X,5HYC(K)
   1,8X,7HPHAC(K),//,(I5,5F15.7))
104 FORMAT (//,4X,6HPUTSQ=,1F15.7,4X,6HERRSQ=,1F15.7,4X,6HONESQ=,1F15.
   17,4X,6HTWOSQ=,1F15.7,/,29X,6HERRSY=,1F15.7,4X,6HONESY=,1F15.7,4X,6
   2HTWOSY=,1F15.7)
105 FORMAT (////,4X,1HJ,11X,4HW(J),7X,8HPHIPP(J),8X,8HPHINN(J),9X,6HYP
   1C(J),///,(I5,4F15.7))
106 FORMAT (1H1,30X,39HDATA LISTED BELOW IS A MEASURE OF POWER,/,
   133X,26HNORMALIZED TO (VOLT)(VOLT),/,33X,24HPHASE DATA IS IN DEGREE
   2S)
107 FORMAT (1H1,30X,31HDATA FOR YP, YC, PHIPP IS IN DB,/,33X,24HPHASE
   1DATA IS IN DEGREES)
108 FORMAT (6X,I2)
110 FORMAT (////,30X,69HGRAPH OF PILOT DESCRIBING FUNCTION VERSUS FREQ
   1UENCY IN RADIANS/SECOND)
111 FORMAT (////,33X,62HGRAPH OF CONTROLLED DYNAMICS VERSUS FREQUENCY
   1IN RADIANS/SECOND)
112 FORMAT (////,27X,75HGRAPH OF PILOT DESCRIBING FUNCTION PHASE VERSU
   1S FREQUENCY IN RADIANS/SECOND)
113 FORMAT (////,30X,69HGRAPH OF CONTROLLED DYNAMICS PHASE VERSUS FREQ
   1UENCY IN RADIANS/SECOND)
114 FORMAT (////,30X,39HGRAPH OF REMNANT POWER VERSUS FREQUENCY)
    COMMON KOUNT,KUIT,C1,D1,CONS1,DELT,T,PI
    COMMON PUT,PUTR,E,ONE,TWO
    COMMON PUTSQ,ERRSQ,ONESQ,TWOSQ,ERRSY,ONESY,TWOSY
    COMMON AMP(29),W(29),A(29),B(29),C(29),D(29)
    COMMON AE(29),BE(29),A1(29),B1(29),A2(29),B2(29)
    COMMON FE(29),FP(29),FC(29)
    COMMON YP(29),PHAP(29),YC(29),PHAC(29)
    COMMON AYP(15),BYP(15),AYC(15),BYC(15),YPC(15)
```

```
      COMMON PHIPP(15),PHINN(15)
      COMMON NREMW,AMPLI
      COMMON U(14,5),JXY(2),U1(13,2)
      CALL DISABLE
      CALL SETPOT (4HP000,.0555,4HP001,.0555,4HP003,.0555,4HP005,.0555)
      CALL SETPOT (4HP030,.2000,4HP031,.2000,4HP033,.1000)
      CALL SETPOT (4HP017,.1600,4HP002,.5000,4HP024,.7500)
S      LDA     $+3
S      STA     052
S      BRJ     45
S      BRM     $+1
S      HLT
      CALL INTP
S      LDA     KUIT
S      BRX     *$-4,0
    4 CALL RESET(1000)
      DELT = 0.02
*     READ IN FREQUENCY OTHER THAN INPUT FOR REMNANT SCALING
      READ(5,108) NREMW
*     FIRST 14 FREQS. GENERATE THE INPUT
*     AMPLITUDE OF LAST 15 FREQS. HAVE VALUE 0.0
      READ(5,100) (AMP(K),W(K),K=1,29)
      WRITE(6,101)(K,AMP(K),W(K),K=1,29)
*     TIME SET SO THAT PROBLEM IS IN STEADY STATE INITIALLY
    1 ONSET = -1080.
*     SCALES INPUT SO THAT NO VALUE GREATER THAN + OR - 0.999
      CONS1 = 0.125
*     SET INITIAL VALUES FOR INPUT,TIME AND ZERO ARRAYS
      PUT = 0.0
      PUTR = 0.0
      CALL DAC(1,PUT,2,PUTR)
      T = 150.
      PI = 3.1415927
*     AMPLI SCALES THE REMNANT AMPLITUDE
      AMPLI = 0.10
```

```
*     DETERMINE ALL FREQS. SO CAN LATER DETERMINE MAGNITUDES OF
*        OUTPUTS AT ALL FREQUENCIES
      DO 10 K=1,29
      A(K) = SIN(W(K)*DELT)
      B(K) = SQRT(1.-A(K)**2)
      C(K) = SIN(W(K)*ONSET)
      D(K) = SQRT(1.-C(K)**2)
      PUT = PUT + AMP(K)*C(K)
   10 CONTINUE
*     GIVES INITIAL VALUE TO DAC
      PUT = PUT*CONS1
      PUTR = AMPLI*C(NREMW)
*     E DENOTES OUTPUT FOR EN
*     1 DENOTES OUTPUT FOR PN
*     2 DENOTES OUTPUT FOR CN
*     INITIALIZE ALL STORAGE AREAS
      DO 20 K=1,29
      AE(K) = 0.
      A1(K) = 0.
      A2(K) = 0.
      BE(K) = 0.
      B1(K) = 0.
      B2(K) = 0.
   20 CONTINUE
      KOUNT = 0
      KUIT = 0
      PUTSQ = 0.
      ERRSQ = 0.
      ONESQ = 0.
      TWOSQ = 0.
*     ENABLE PROBLEM TO BE STARTED WHEN OPERATOR READY
      OUTPUT(101) !SET DSO SWITCH TO START RUN!
  511 IFLG1 = TEST(3)
      IF (IFLG1.GT.0) GO TO 511
      CALL COMPUTE
```

```
      CALL ENABLE
   30 CONTINUE
      IF (KUIT.EQ.1) GO TO 31
      GO TO 30
   31 CONTINUE
      CALL DISABLE
      CALL HOLD
      CALL ACK(4,PUTSQ,5,ERRSQ,6,ONESQ,7,TWOSQ)
      CALL RESET(1000)
      WRITE(6,102) (K,AE(K),BE(K),A1(K),B1(K),A2(K),B2(K),K=1,29)
      ERRSQ = ERRSQ*4.
      ERRSY = ERRSQ/PUTSQ
      ONESY = ONESQ/PUTSQ
      TWOSY = TWOSQ/PUTSQ
*     RMSCON SCALES THE ROOT MEAN SQUARE VALUE
      RMSCON = 1000.
      PUTSQ = PUTSQ*RMSCON
      ERRSQ = ERRSQ*RMSCON
      ONESQ = ONESQ*RMSCON
      TWOSQ = TWOSQ*RMSCON
*     FE IS NOTATION FOR EN
*     FP IS NOTATION FOR PN
*     FC IS NOTATION FOR CN
      DO 40 K=1,29
      FE(K) = AE(K)**2 + BE(K)**2
      FP(K) = A1(K)**2 + B1(K)**2
      FC(K) = A2(K)**2 + B2(K)**2
      YP(K) = SQRT(FP(K)/FE(K))
      YC(K) = SQRT(FC(K)/FP(K))
      PHAP(K) =57.3*(ATAN(BE(K),AE(K)) - ATAN(B1(K),A1(K)))
      PHAC(K) = 57.3*(ATAN(B1(K),A1(K)) - ATAN(B2(K),A2(K)))
*     ASSUME PHASE LEAD LESS THAN 180 DEGREES TO CORRECT FOR LOSS OF
*        PHASE INFORMATION IN ARC-TANGENT ROUTINE
      IF (PHAP(K).LT.180.) GO TO 42
      PHAP(K) = PHAP(K) - 360.
```

```
42 IF (PHAC(K).LT.180.) GØ TØ 44
   PHAC(K) = PHAC(K) - 360.
44 IF (PHAP(K).GT.-180.) GØ TØ 41
   PHAP(K) = 360. + PHAP(K)
41 IF(PHAC(K).GT.-180.) GØ TØ 43
   PHAC(K) = 360. + PHAC(K)
43 CØNTINUE
40 CØNTINUE
   CØNS2 = 0.046757339
*     DETERMINE REAL AND IMAGINARY PØRTIØNS ØF YP AND YC
   DØ 45 K=1,14
   AYP(K) = YP(K)*CØS(PHAP(K)/57.3)
   BYP(K) = YP(K)*SIN(PHAP(K)/57.3)
   AYC(K) = YC(K)*CØS(PHAC(K)/57.3)
   BYC(K) = YC(K)*SIN(PHAC(K)/57.3)
45 CØNTINUE
*     PØLATE IS FACTØR USED FØR INTERPØLATIØN ØF REMNANT SIGNAL
   DØ 46 K = 1,13
   J = K + 15
   PØLATE = (W(J) - W(K))/(W(K+1) - W(K))
   YPC(K) = (1. + (((AYP(K) + AYP(K+1))*PØLATE) * ((AYC(K) + AYC(K+1
  1)*PØLATE)) - (((BYP(K) + BYP(K+1))*PØLATE) * ((BYC(K) + BYC(K+1))*
  2PØLATE)))**2  +  ((((BYP(K) + BYP(K+1))*PØLATE) * ((AYC(K) + AYC(K
  3+1))*PØLATE)) + (((AYP(K) + AYP(K+1))*PØLATE) * ((BYC(K) + BYC(K+1
  4))*PØLATE)))**2
   PHIPP(K) = FP(J)/(2.*T)
   PHINN(K) = PHIPP(K)*YPC(K)*CØNS2
46 CØNTINUE
   WRITE (6,106)
   WRITE(6,103) (K,W(K),YP(K),PHAP(K),YC(K),PHAC(K),K=1,14)
   WRITE (6,105) (J,W(J+15),PHIPP(J),PHINN(J),YPC(J),J=1,13)
   WRITE(6,104) PUTSQ,ERRSQ,ØNESQ,TWØSQ,ERRSY,ØNESY,TWØSY
   DØ 50 K=1,14
   U(K,1) = ALØG10(W(K))
   U(K,2) = 20.*ALØG10(YP(K))
```

```
      U(K,3) = 20.*ALOG10(YC(K))
      U(K,4) = PHAP(K)
      U(K,5) = PHAC(K)
   50 CONTINUE
      DO 51 K = 1,13
      J = K + 15
      U1(K,1) = ALOG10(W(J))
      U1(K,2) = 20.*ALOG10(PHINN(K))
   51 CONTINUE
      WRITE (6,107)
      WRITE (6,103) (K,W(K),U(K,2),PHAP(K),U(K,3),PHAC(K),K=1,14)
      WRITE (6,105) (J,W(J+15),PHIPP(J),U1(J,2),YPC(J),J=1,13)
      JXY(1) = 1
      JXY(2) = 2
      CALL VPLOT(U,JXY,14,14,1,1,-2.,2.,-60.,60.)
      WRITE (6,110)
      JXY(2) = 4
      CALL VPLOT(U,JXY,14,14,1,1,-2.,2.,-180.,180.)
      WRITE (6,112)
      JXY(2) = 3
      CALL VPLOT(U,JXY,14,14,1,1,-2.,2.,-60.,60.)
      WRITE (6,111)
      JXY(2) = 5
      CALL VPLOT(U,JXY,14,14,1,1,-2.,2.,-180.,180.)
      WRITE (6,113)
      JXY(2) = 2
      CALL VPLOT(U1,JXY,13,13,1,1,-2.,2.,-60.,60.)
      WRITE (6,114)
*     ENABLES PROBLEM TO START AGAIN
*     IN ORDER TO STOP PUSH IDLE THEN RESET ON CONSOLE
      OUTPUT(101) 'SET DS1 SWITCH TO INITIALIZE FOR ANOTHER RUN'
  512 IFLG2 = TEST(4)
      IF (IFLG2.GT.0) GO TO 512
      GO TO 1
    3 STOP
```

```
      SUBRØUTINE INTR
      CØMMØN KØUNT,KUIT,C1,D1,CØNS1,DELT,T,PI
      CØMMØN PUT,PUTR,E,ØNE,TWØ
      CØMMØN PUTSQ,ERRSQ,ØNESQ,TWØSQ,ERRSY,ØNESY,TWØSY
      CØMMØN AMP(29),W(29),A(29),B(29),C(29),D(29)
      CØMMØN AE(29),BE(29),A1(29),B1(29),A2(29),B2(29)
      CØMMØN FF(29),FP(29),FC(29)
      CØMMØN YP(29),PHAP(29),YC(29),PHAC(29)
      CØMMØN AYP(15),BYP(15),AYC(15),BYC(15),YPC(15)
      CØMMØN PHIPP(15),PHINN(15)
      CØMMØN NREMW,AMPLI
      KØUNT = KØUNT+1
*     FIRST 30 SECØNDS IS WARM-UP TIME
      IF (KØUNT.GT.9000) GØ TØ 30
*     NEXT 150 SECØNDS IS RUN TIME
      IF (KØUNT.GT.1500) GØ TØ 20
      CALL DAC(1,PUT,2,PUTR)
*     FRØM THIS PØINT TØ VGØ TØ 32V IS WARM-UP PØRTIØN
      PUT = 0.0
      DØ 10 K=1,29
      C1 = A(K)*D(K) + B(K)*C(K)
      D1 = B(K)*D(K) - A(K)*C(K)
      C(K) = C1
      D(K) = D1
      PUT = PUT + AMP(K)*C1
   10 CØNTINUE
      PUT = PUT*CØNS1
      PUTR = AMPLI*C(NREMW)
      GØ TØ 32
*     THIS PØRTIØN IS RUN PØRTIØN
  20. CALL ADCA(E,3,PUT,2)
      PUT = 0.0
      DØ 21 K=1,29
      C1 = A(K)*D(K) + B(K)*C(K)
      D1 = B(K)*D(K) - A(K)*C(K)
```

```
      C(K) = C1
      D(K) = D1
      IF (K.LE.14) PUT=PUT+AMP(K)*C1
      AE(K) = AE(K) + E*D1
      A1(K) = A1(K) + ØNE*D1
      A2(K) = A2(K) + TWØ*D1
      BE(K) = BE(K) + E*C1
      B1(K) = B1(K) + ØNE*C1
      B2(K) = B2(K) + TWØ*C1
   21 CØNTINUE
      PUT = PUT*CØNS1
      PUTR = AMPLI*C(NREMW)
      GØ TØ 32
*     KUIT SETS PRØGRAM END TIME--TAKES IT ØUT ØF INTERRUPT CØNTRØL
   30 KUIT = 1
   32 RETURN
      END
```

The following subroutine calls analog-to-digital conversion and vice versa to conserve computer time between interrupts.

```
X1      EQU     1
X2      EQU     2
A       EQU     5
B       EQU     4
$ADDA   PZE
        BRM     9SETUPN
        PZE     4
AD      PZE
NAD     PZE
DA      PZE
NDA     PZE
        SKR     FINFLG
        BRU     $-1
        LDA     BRMFIN
        STA     040
        STA     041
        LDA     *NAD
        STA     COUNT
        SKR     COUNT
        BRU     $+2
        BRU     DTOA
        LLSA    15
        ADD     =ADC8M
        STA     ADCW
        SKN     FINFLG
        BRU     $-1
        STZ     FINFLG
        EOM     034001
        POT     ADCW
        LDX     =00100000,1
        SKN     FINFLG
        BRU     $-1
ITER    LDA     ADBUF,1
        COPY    (0,3)
        FLA     0.0
```

```
        STD     *AD
        MPT     AD
        SKR     CØUNT
        BRX     ITØR,1
DTSA    LDA     *NDA
        MRG     =C77700000
        CØPY    (A,Y1)
        CØPY    (C,A)
        STA     DACØM,1
        BRX     $+2,1
        BRR     ADDA
        STZ     DCH
        BRJ     $+2
RTØI    MPT     DA
        MPØ     DCH
        LDP     *DA
        CRSD    9
        ARSD    15
        CØPY    (-A,A)
        CØPY    (A,X2)
        CØPY    (E,A)
        ARSA    C,2
        ETR     =077777000
        ADD     DCH
        STA     DACØM,1
        BRX     RTØI,1
        STZ     FINFLG
        LDA     *NDA
        LLSA    15
        ADD     =DACØM
        STA     DACW
        EØM     035001
        PØT     DACW
        BRR     ADDA
DAMFIN  BRM     FIN
```

```
FIN     PZE
        SKR     FINFLG
        BRU     $-1
        BRC     *FIN
FINFLG  PZE
OCH     PZE
COUNT   PZE
CRN     FORM    9,15
ADCA    PZE
ADCOM   CRN     0,ADBUF
        DATA    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
        DATA    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
ADBUF   RES     32
DACA    PZE
DACOM   RES     13
        END
```

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13. ABSTRACT

The measurement of the human describing function and remnant in a compensatory tracking task is undertaken. These measurements are obtained through the application of the fast Fourier transform technique on a hybrid (analog-digital) computer. This method processes the data in real time with minimal core storage and the results are available immediately upon completion of the tracking run.

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2. Human Response Function						
3. Human Transfer Function						
4. Pilot Response						
5. Describing Function						
6. Human Remnant						
7. Remnant						

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